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Computer routines were devised to investigate the behavior of two statistics, the Student t and the Mann-Whitney U, under specific conditions. Certain of these conditions were violations of the underlying assumptions of the two tests. Both were computed for 5,000 pairs of samples sized 5, 10, and 20 from a rectangular, a normal and an exponential distribution. The scales of measurement were progressively changed from those as continuous as possible to those of 50 cells, 20 cells, 10 cells, 4 cells, 3 cells, and 2 cells. For the Mann-Whitney U statistic, ties were handled by (1) putting all tied members of sample number one before those in sample number two tied for the same position, (2) assigning members of a tie group the mean of the ranks for which they were tied, and (3) assigning each member of a tied group to a random number of the positions for which they were tied. Results for the Student t statistic varied little from the tabled results even for a dichotomous scale. The Mann-Whitney U statistic, however, was very heavily affected by discreteness. (Author/TT)

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Effects of Inapplicability of the Continuity Condition upon the Probability Distributions of Selected Statistics and their Implications for Research in Education

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Jack N. Sparks

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U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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INTRODUCTION

Most of the statistical procedures currently in use in educational and psychological research require that the sampled populations be continuously distributed on the trait measured. This condition holds rather generally for both parametric and nonparametric statistics. The distributions used as models for most parametric test statistics are continuous, increasing or decreasing smoothly, without fluctuation, except for a possible change of direction at a single mode. The corresponding condition for nonparametric or "distribution-free" statistics is usually more simply that sampled populations be continuously distributed, although sometimes a further condition is added--that the sampled populations have identical shape or distributions symmetrical about the same point.

A continuously distributed population has an infinite number of abscissae and thus an infinite number of different score magnitudes, each having zero *a priori* probability of being drawn upon any occasion of sampling. Theoretically, therefore, a sample from a continuously distributed population will contain no scores of zero and no tied scores, since zero is a predesignated score and since the first-drawn member of a tied group can be considered to predesignate the remainder. Zero scores are thus embarrassing in tests using the algebraic sign of scores, and tied scores are undersirable in tests which rank scores and whose derivation requires that each rank occur only once.

In most educational and psychological research, however, the continuity condition seems unrealistic. Even if the sampled population is continuously distributed on the trait measured, measurements made upon members of that population must be discretely distributed, since no measuring instrument is capable of infinite precision. Thus, if any population of actual magnitudes is transformed into measurements on a scale running from zero to one with precision possible to the N^{th} decimal place, then the population of measurements is a discrete population whose interval width is the difference between successive digits at the N^{th} decimal place. In every case, therefore, the actual distribution of measurements violates the continuity condition. Further, the gross categorizations often encountered in social research make the violation of the continuity condition rather glaring. Consider, for example, the comparison of the responses of samples from two different populations to a values inventory item offering five choices along a continuum. If the Kolmogorov-Smirnov maximum deviation test for identical populations is applied, the large number of tied scores is extremely embarrassing, apparently invalidating the application. Similar illustrations may be given for most other common nonparametric tests.

Purposes

The general matter of concern in this study is the extent to which the probability distributions of commonly computed statistics vary from their theoretical models as a result of violations of the continuity condition to varying degrees.

The answer to the above problem cannot presently be derived on the basis of a theoretical mathematical development, because of the infinite variety of degrees of discreteness possible. Some enlightenment can be obtained, however, by sampling from a distribution of measures as nearly continuous as possible. Successive samplings can then be obtained from distributions based on increasingly gross categories of the first distribution. Results obtained in preliminary work with the Mann-Whitney U-test and the Student t-test indicated that there would be some changes in the sampling distributions as discrete categories become broader and fewer.

In the investigation reported herein, those sampling studies were extended to additional distributions and additional test procedures.

The specific objectives of the study were:

1. To observe the behavior of the distributions of selected nonparametric tests as distributions of measures are made successively more discrete.
2. To compare the empirically obtained distributions of the selected statistics with each other and with the t-test.
3. To draw inferences concerning efficiency and validity from the results.
4. To provide evidence to serve as a basis for suggestions to assist educational researchers in making decisions on choices of procedures for comparison of treatment groups.
5. To isolate promising lines of attack for exploring the problem further.

Review of Related Research

Several investigators have reported studies dealing with the robustness of certain parametric tests and their sensitivity to violations of various conditions. Considerable attention has been focused on the t and F statistics (Norton, 1952; Box, 1955, 1962; Srivastava, 1959; Boneau, 1960). In empirical studies these two tests procedures have been found to be quite insensitive to violations of the conditions of normality of distribution and homogeneity of variance. The fact is, however, that any violation of conditions can be expected to alter the distribution of the test statistic and change the value at which the test statistic becomes significant. Whether or not this effect is negligible depends not only on the degree to which the condition is violated, but also upon other factors such as sample size and significance level.

Recently numerous theoretical papers have compared power efficiencies and asymptotic relative efficiencies of certain parametric and nonparametric tests, under the conditions of the parametric tests (Stuart, 1952; van der Waerden, 1952; Lehmann, 1953; Dixon, 1954; Mood, 1954,; Stuart, 1954;

Hodges & Lehmann, 1956; Mikulski, 1963; Pratt, 1964; Chanda, 1963). Under these conditions, of course, the theoretical power curves of the non-parametric procedures run quite consistently under those of their parametric analogues, and the asymptotic relative efficiencies of nonparametric procedures never exceed .955. Rather typical of the findings reported in these papers are those of Lehmann (1953) who examined a number of statistical tests and reported the following order of decreasing power of these tests: Student's t-test, Mann-Whitney U-test, Westenberg median test, maximum absolute deviation test, and number of runs test.

Empirical studies of the relative powers of parametric and nonparametric tests under various conditions have not yielded results essentially different from the theoretical works. Dixon and Teichroew (1954), sampling from normal distributions with homogeneous variances, found that the nonparametric tests they examined ranked as follows in order of power: Mann-Whitney U-test, maximum absolute deviation test, median test, run test. This is as predicted. (See Lehmann, 1953, above). Boneau (1962) compared the Mann-Whitney U-test with the t-test on samples from four different populations (normal with homogeneous variance, normal with heterogeneous variance, rectangular and expotential) using several different sample sizes. He found the U-test to be sometimes more sensitive to distribution differences than the t-test. In all comparisons, however, the powers of the two tests were approximately the same. Epstein (1955) also drew samples from standardized normal distributions (N = 10) and calculated values for the Mann-Whitney U-test, the runs test, the maximum deviation test, and an exceedance test. His results with respect to power were essentially the same as those of Dixon.

All of the above investigations retained the condition of continuous distributions, although other departures from the conditions of the parametric tests were sometimes made. A few investigators have examined to a limited extent the effect of discreteness on the power and other characteristics of nonparametric tests (David, 1952; Putter, 1955; Tate, 1956; Durbin, 1961; Chanda, 1963; Slakter, 1965). Tate sampled from a normal distribution (population N = 400) with a range of whole number scores from 11 to 71. From 100 sample pairs of size 16 and 9 he calculated values for the t-test, Mann-Whitney U-test, median test and runs test. Experiments were run under four conditions: a) the differences as observed, b) the differences after 4 had been added to each of the original scores in samples of size 9, c) the differences after 8 had been added to each of the original scores in samples of size 9, and d) the differences after 12 had been added to each of the original scores in samples of size 9. A single method of breaking ties was used for each test. For the Mann-Whitney U-test, ties were broken by assigning to each tie the average of the serial ranks tied for. Ties at the median were broken at random in the case of the median test. When the runs test was calculated, ties were ordered at random. Under these conditions, the Mann-Whitney U-test apparently lost very little power and was nearly as powerful as the parametric procedure.

Putter's (1955) work on the asymptotic efficiency of the Wilcoxon (Mann-Whitney) statistic demonstrated that problems of handling ties are not easily overcome. One important conclusion of his examination of the problem was the discovery that although random assignment of ties allows application of the Wilcoxon probability tables from a mathematical standpoint, the randomization procedure results in reduced efficiency. As far as this method of handling ties is concerned there is also the common objection that the outcome of the test is made to depend not only on the observations, but also on an additional, and more or less irrelevant, random experiment. Putter worked out some solutions for asymptotic power of the Wilcoxon with a non-randomization procedure. In a similar approach, Chanda (1963) was concerned about the efficiency of non-parametric tests when the distributions are discrete relative to the most powerful tests from parametric families for such distribution. He worked out the asymptotic power efficiency of the Mann-Whitney statistic for the class of discrete distributions which are of the exponential type.

No work is known to this investigator which faced squarely the problem in the case of finite sample size except that conducted with the Kolmogorov statistic. Durbin (1961) noted that the Kolmogorov test has good asymptotic power against alternatives specified in terms of distance between distribution functions, but in practice, with samples of moderate size, it has frequently failed to give a significant result when the chi square test has done so. He explained that if the alternative distribution is such that the difference between the hypothetical distribution function and the alternative distribution function is relatively large for some given value of the variate, the Kolmogorov test would almost certainly detect the difference. However, if the difference between the two distribution functions is widespread but nowhere large (as is possible if their means and variances are the same even though the two frequency functions differ markedly in shape), the Kolmogorov statistic is not likely to be a powerful discriminator. Durbin followed up with empirical results on the Kolmogorov test, the chi square test, a modified Kolmogorov test, and a modified median test. Experimental data were based upon five samples of size 50 from exponential, Laplace, and normal distributions. The hypothesis that the true distribution was normal was then tested. His limited data seemed to support the contention outlined above. A more compelling reason for the poor performance of the Kolmogorov test in common practice is seen, however, in Slakter's (1965), empirical study. He took into account the fact that in goodness of fit applications not only is the number of groups finite, thus making data discrete, but the number of categories is often quite small indeed. Random samples were taken from equal probability groups, with several combinations of sample size between 10 and 50 and number of groups between 5 and 50. Under these conditions the Pearson chi square goodness of fit test retained robustness of validity, even when the sample size and number of groups was equal, say 50 and 50. On the other hand the Kolmogorov test was more conservative than predicted, indicating that statements of comparative validity of these two test procedures might well be revised. Slakter's study was limited, however, to sampling from equal probability categories, so that further work is required to investigate this problem thoroughly. Further, no attention was given to power.

The principal investigator in the study reported herein has, since 1965, been pursuing exploratory sampling studies dealing with the discontinuity problem. Initially, computer problems were constructed to draw pairs of samples of several sizes from a rectangular distribution (generated by computer), to make these observations more and more "discrete," and to compute the Student t-statistic and the Mann-Whitney U-statistic for each pair of samples under each degree of discreteness. The t-statistic was included as a parametric procedure much investigated against which results with non-parametric procedures could be compared, while the choice of the Mann-Whitney as a first non-parametric test to investigate was purely arbitrary. Those initial programs were designed to provide 100 "t's" and "U's", and the results were uch as to encourage the present work, because they seemed to indicate that the U-test deteriorated somewhat faster than the Student t-test as discreteness was increased.

METHOD

As a preliminary step in the investigation of the discontinuity problem for nonparametric statistics, it was necessary to develop a series of computer programs. The first set of such programs allowed the drawing of pairs of samples from (a) a rectangular, (b) a normal, and (c) an exponential-type distribution. These were followed by a program which allowed the range of the scale of values of the samples to be divided into any set number of categories, thus allowing control of the degree of discreteness of measures to which statistical tests were to be applied. Subroutines for the statistical tests were then inserted, and followed where appropriate by the several subroutines for various ways of handling ties. Finally, subroutines for developing frequency distributions were included. Listing of all programs are presented as Appendix A.

Samples of five thousand of each of the statistics under investigation were obtained under the various specified conditions. These samples were all made for "uniformity trials," that is, assuming no displacement between groups. Thus, no power studies were done, and comparisons reported are simply straight forward comparisons of statistics computed on distributions having the same central tendency.

RESULTS

The Student t Statistic

a. Sampling from a normal distribution.

Results are shown in Appendix B, Tables 1, 2, and 3.

Obtained proportions are given in the tables for values of t at certain critical probabilities of the t distribution. In addition to the continuous case, results for seven degrees of discreteness, ranging from a division of the range of values into 50 cells to a division into 2 cells are given. For samples of size 20, (the largest samples drawn) it may be seen that even the continuous case contains some deviation from the expected proportions. This may indicate that our sampling routine did not truly return random normal deviates. As far as the effect of discreteness is concerned, however, there appears to be little effect, with proportions close to those of the continuous case. Much the same can be said for the results from samples of size 10 (Table 2) and size 5 (Table 3). In all three tables one peculiarity which we could not interpret appeared--the tail proportions were lower when the scale of values was discreted to only three cells than for any other degree of discreteness.

b. Sampling from a rectangular distribution.

Tables 4, 5, and 6 present proportions for t's calculated on samples from a rectangular distribution. These differ a bit from the tabled probabilities for the t distribution, especially at sample size 5, but the proportions are nontheless remarkably close to the tabled values--even when the measuring scale has only two categories.

c. Sampling from an exponential-type distribution.

For samples of size 5 proportions at the extreme tails were quite distorted, with very low proportions at the lower end, up to an expected proportion of .01, and higher proportions at the upper end. This held true for the continuous case and through discreting down to a scale with only five categories. Beyond that there began to be also too few at the upper end.

With increasing sample size, however, the distributions of obtained t's approach much more closely the tabled distributions. By the time sample size is increased to 20, the distributions are extremely close to the theoretical one. The relative frequency values in the tails do appear to decrease as discreteness is increased, however, and when the measuring scale has only two categories, the tail probabilities are much too small.

In summary, the sampling studies conducted with the Student t statistic demonstrate its stability under many kinds of violations of its underlying conditions. In this study we have found it to be quite stable under violations of the continuity conditions.

Mann-Whitney U Statistic

a. Sampling from a rectangular distribution.

For samples of size 5 and ties handled by putting all tied members of sample number two tied for the same position, there was little change until the score distribution had less than five cells, although the cumulative relative frequencies for the smaller values of U increased progressively as discreteness increased. (Table 9). For degrees of discreteness beyond that point, however, cumulative relative frequencies for the smaller values of U soon became several times greater than the critical probabilities. When ties were handled by assigning each member of a tied group to a random one of the positions for which they were tied, there was very little change in cumulative relative frequencies from the continuous case to the case of a two cell measuring scale (Table 10).

The result obtained when ties were handled by assigning members of a tie group the mean of the ranks for which they were tied was just opposite to the result obtained under the first condition for handling ties (Table 11). Cumulative relative frequencies for smaller values of U decreased progressively as discreteness increased.

In Tables 12-14 are reported the results for samples of size 10 from a rectangular distribution. With ties handled in the first way, the results for samples of size 5 are practically duplicated, with cumulative relative frequencies piling up even faster for smaller values of U (Table 12). When ties are handled by the random method, the results for the continuous case and the two cell case are practically identical (Table 13), with only random variation between. This result is again in line with results obtained for samples of size five.

For the case in which ties were handled by the mean method (Table 14), however, a strange reversal happened for this sample size. Contrary to the results obtained for samples of size 5, we found that cumulative relative frequencies for the smaller values of U became progressively less as discreteness increased.

Results for samples of size 20 are given in Tables 15-17. Those for the first method of handling ties (Table 15) demonstrate essentially the same phenomenon observed earlier for smaller sample sizes, except that deviations from critical probabilities do not become extreme quite so soon. Nevertheless, by the time the measuring scale is down to five categories the cumulative relative frequencies at the lowest end are more than ten times the expected values.

Again (Table 16), when ties are handled by the random procedure, cumulative relative frequencies for the continuous case and all degrees of discreteness are very like the critical probabilities of all levels.

There appears to be some settling down for the case of ties handled by the mean method (Table 17) by the time samples have reached a size of 20. The cumulative relative frequencies deviate only slightly from critical probabilities for all degrees of discreteness except the least. When the measuring scale has only 2 cells (the dichotomous case) cumulative relative frequencies for the smaller values of U are considerably less than the expected ones.

b. Sampling from an exponential distribution.

1. Samples of size 5

In the continuous case the distribution for this situation was much the same as the theoretical or expected one. When ties were handled by putting all tied members of sample number one before those in sample number two tied for the same position, the cumulative probabilities for the smaller values of U increased progressively as discreteness increased (Table 18). This result was particularly marked for conditions of three and two category measuring scales.

When ties were handled by the random method, there was very little deviation from the continuous case for any degree of discreteness. All cumulative relative frequencies were similar to the theoretical values (Table 19).

When, on the other hand, ties were handled by the mean method, the results were very different. There was little change from the continuous case until discreteness had reached the point where there were less than five categories on the measuring scale (Table 20).

2, Samples of size 10

At this sample size the continuous case showed cumulative relative frequencies close to the theoretical ones only for probability .01 and higher. Values with lower probability of occurrence did not appear (Table 21). When ties were handled by putting all tied members of sample number one before those in sample number two tied for the same position, there was little change from the continuous case until the measuring scale was down to less than five categories. From that point down to the two category situation, smaller values of U appeared much more frequently than would be expected.

When ties were handled by the random method, there was a somewhat irregular deviation from the continuous case as discreteness increased until at the point at which the measuring scale was dichotomous, the cumulative relative frequencies were very much like the theoretical ones (Table 22).

For the situation in which ties were handled by the mean method, the results for measuring scales of 50, 20, and 10 cells were very little different from the continuous case (Table 23). Beyond that part, however, the cumulative relative frequencies for the smaller values of U decreased progressively as discreteness increased. The result was, in essence, a reversal of that obtained by the first method.

3. Samples of size 20.

Cumulative relative frequencies obtained for the continuous case were greater for the extremely small values of U than are the theoretical probabilities. Actually, what happened was that .01 of the U values were smaller than the .001 value, but no more occurred until the .01 level was reached. From that point cumulative relative frequencies continued to run slightly higher than expected until the middle of the distribution was reached. Handling ties by putting all tied members of sample number one before those in sample number two tied for the same position resulted in no great change until the measuring scale was down to five cells (Table 24). As discreteness increased from that point, smaller values of U occurred with progressively greater relative frequency as discreteness increased. It should be noted that when the measuring scale became dichotomous, more than half of the occurring U values were less than the value below which we would expect only .001 to occur.

When ties were handled by the random method, the only change from the continuous case as discreteness increased was a change to a distribution more like the theoretical one (Table 25). For a measuring scale of only two cells the distribution of cumulative relative frequencies was almost identical to the expected distribution of U's.

The table of cumulative relative frequencies for the situation in which ties were handled by the mean method (Table 26) reveals results consistent with previous applications of this method. As discreteness increased to the point where the measuring scale had less than five cells, small values of U had smaller and smaller cumulative relative frequencies.

c. Sampling from a normal distribution.

Technical difficulties still unresolved in connection with our computer facility make it impossible to include in this report the results for handling ties by the mean method. Consistency of the results below with all others previously reported, however, lead us to believe that no conclusion of the report would be changed were these missing results available.

1. Samples of size 5

In Table 27 it may be seen that the previously obtained results for handling ties by the method of putting all tied members of sample number one before those in sample number two tied for the same position are duplicated. As discreteness increased, cumulative relative frequencies of the smaller values of U increased. One puzzling aberration appeared, however, which was consistent for all sample sizes, becoming more pronounced for the larger samples. The greatest deviation from theoretical probabilities occurred when the measuring scale was cut to three cells. Cumulative relative frequencies for this situation were extremely high for the smaller values of U. For the two cell measuring scale the cumulative relative frequencies of smaller values of U were only slightly larger than for the four cell measuring scale.

When ties were handled by the random method (Table 28), results for all degrees of discreteness were almost identical to the theoretical probabilities.

2. Samples of size 10

Again, as illustrated in Table 29, the first method of handling ties produced increasing cumulative relative frequencies for smaller values of U as discreteness increased. The phenomenon noted for a measuring scale of three cells at sample size five is even more pronounced at this sample size.

Handling ties by the random method produced (Table 30) results similar to the continuous case for all degrees of discreteness. All of the cumulative relative frequencies were close to the theoretical probabilities.

3. Samples of size 20

Results of handling ties by the first method were similar to those obtained with other sample sizes, with the tendencies to greater cumulative relative frequencies for small values of U even more pronounced (Table 31).

The most extreme results for the measuring scale of three cells were obtained at this sample size.

Handling ties by the random method (Table 32) again produced consistent results for the continuous case and all degrees of discreteness, with no marked differentials. There did seem to be a slight tendency for the cumulative relative frequencies of the very smallest values of U to become progressively smaller as discreteness increased.

DISCUSSION

By and large the results present a consistent picture, with the Student t statistic hardly disturbed at all by discreteness. Disturbances in the distribution of the Mann-Whitney U statistic depended upon the method used for handling ties, but were quite consistent across sample sizes and sampled distributions.

Both statistics were most disturbed from their expected values when sampling was from an exponential distribution and sample sizes were small. Because of the relative stability of the Student t statistics under all of the violations of its conditions which were posed in this study, the discussion must concentrate on the Mann-Whitney U statistic.

Only in the case of sampling from an exponential distribution did the cumulative relative frequencies for U's, in the continuous case, deviate from the theoretical probabilities to any considerable extent. The differences in cumulative relative frequencies due to discreteness turned out to be predictable ones dependent upon the method of handling ties.

When ties were handled by putting all tied members of samples number one before those in sample number two tied for the same position, the inevitable result was greater cumulative relative frequencies for the smaller values of U. It would seem off-hand that this would produce higher proportion of rejections of the null hypothesis than desired, but this procedure is generally used as an attempt at conservatism and the assignment is made in the way least conducive to rejection. It therefore seems likely that such a situation, involving a one-ended test, would result in the extra extreme values being ignored, since they are in the wrong direction, truly producing a conservative test.

When ties were handled by assigning the members of a tie group the mean of the ranks for which they were tied, the tendency with increasing discreteness was invariably toward smaller cumulative relative frequencies for the smaller values of U. This method will produce fewer rejections of the null hypothesis. It might be said that this is indeed a "conservative" procedure for handling ties.

The method of handling ties which is commonly called the only mathematically defensible one turned out to produce consistent results. In these situations of drawing from two identical distributions, handling ties by assigning each member of a tied group to a random number of the positions for which they were tied resulted in cumulative relative frequency distributions very similar to the theoretical probabilities regardless of the degree of discreteness.

CONCLUSIONS, IMPLICATIONS, RECOMMENDATIONS

Since the Student t is more stable than the Mann-Whitney U under all of the conditions investigated, it is certainly a preferred procedure. When the Mann-Whitney U statistic is to be used, the random method of handingly ties is to be preferred.

SUMMARY

Computer routines were devised to investigate the behavior of two statistics, the Student t and the Mann-Whitney U, under specific conditions, certain of which were violations of underlying assumptions. Specifically, both were computed for 5,000 pairs of samples (of sizes 5, 10, and 20 in turn) from a rectangular, a normal, and an exponential distribution. The scales of measurement were progressively changed from as continuous as possible to scales of 50 cells, 20 cells, 10 cells, 5 cells, 4 cells, 3 cells, and 2 cells. For the Mann-Whitney U statistic, ties were handled by three different methods:

1. By putting all tied members of sample number one before those in sample number two tied for the same position.
2. By assigning members of a tie group the mean of the ranks for which they were tied.
3. By assigning each member of a tied group to a random number of the positions for which they were tied.

Results for the Student t statistic varied little from the tabled results even for a dichotomous scale. The Mann-Whitney U statistic, however, was very heavily affected by discreteness when the first and second methods of handling ties were used. When the random method of handling ties was used, the Mann-Whitney statistic for even dichotomous scales yielded cumulative relative frequency distributions much like those for continuous scales.

If the Mann-Whitney U statistic is to be used, it would seem that the random method of handling ties is the logical choice.

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```
C          ..........................................................
C                                                                 A 1
C          * * * * * * * * *  P R O G R A M   S I D O T  * * * * * * * * *
C                            * * * * * * * * * * * * * *
C          ..........................................................
C
C
C
C                               D A T A   C A R D S
C                               ===================
C
C
C
C     THE LAST DATA CARD MUST BE A BLANK
C
C
C
```

```
C         ------------------ S P E C I F I C A T I O N S ------------------
C                            ----------------------------
C
C
C
      LOGICAL*1OUTPUT(129)
      LOGICAL*1PER/'.'/, BLANK/' '/, AST/'*'/
      LOGICAL*1RUNTYP(15)
      INTEGERCASENO
      INTEGER COUNT  (200)
      INTEGER BASSET
      INTEGERSAMSIZ,BATCH
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONX(100,20),Y(100,20),T(100,5),IBASE(2),JBASE(2),PRNSAM,PUNSAM
     1,PRINTT,PUNCHT,CONTRL,NOBATS,WRITET,WRISAM,ABSMIN,NOSAMS,SAMSIZ,SC
     1OMAX,NOCELL(5),TEST      ,KBASE(2)
      COMMON MEAN,VAR
C
C
C
C         --------- S E T   I N I T I A L   C O N D I T I O N S -------
C                   ---------------------------------------------
C
C
      CALL SETBTF
      PRINT90025
C
      NUMTIM=0
      CASENO=0
```

```
C
C                     R E A D   I N P U T   D A T A
C                     ............................
C
C
90026 READ90027 ,NOSAMS,SAMSIZ,SCOMAX,(NOCELL(I),I=1,5)
      IF(NOSAMS) 3010,3010,3011
 3010 STOP
 3011 READ 90022, TEST,PRNSAM,PUNSAM,PUNCHT,CONTRL,NOBATS,PRINTT,WRITET
C
C
      READ90028 ,IBASE(1),IBASE(2),IBEG,IEND
 2112  CONTINUE
 2111 KBASE(1)=IBASE(1)
      KBASE(2)=IBASE(2)
      READ 2100,N,FMAX,FMIN,NUMCO    ,ISCAL
 2100 FORMAT(1(8X,I2),2F10.0,2(7X,I3))
      READ 3001,MEAN,VAR
 3001 FORMAT(2F10.3)
      VAR=SQRT(VAR)
      Z=FMAX-FMIN
      FINTER    =Z/FLOAT(NUMCO    )
      PRINT 2000,FMIN,FMAX,FINTER
 2000 FORMAT('1THE MINIMUM SAMPLE IS ',1PE14.7,'.  THE MAXIMUM SAMPLE IS
     1 ',E14.7,'.  THE STEP USED IS ',E14.7)
      PRINT 2099
 2099 FORMAT('0THE FOLLOWING ARE THE INTERVALS USED AND THEIR RANGES.'/'
     10',T27,'INTERVAL',T45,'LOWER LIMIT',T66,'UPPER LIMIT')
       II=NUMCO    -1
      RANGE=FMIN
      DO 2001 M=1,II
      XX=RANGE
      YY=XX+FINTER
      RANGE=YY
 2001 PRINT 3000,M,XX,YY
 3000 FORMAT('0',T29,I4,T44,1PE14.7,T65,E14.7)
      M=NUMCO
      XX=FMAX
      PRINT 3000,M,YY,XX
C
      READ90022 ,ABSMIN
C
C
C
C                         I S   R U N   C O M P E T E D
C                         .............................
C
C
      IF(CONTRL)90029 ,90030 ,90029
90030 PRINT90031
      IF(WRITET)90032 ,90033 ,90032
90032 CONTINUE
90033 CONTINUE
      PRINT90034
      STOP
C             $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $
C
C
C
C                              H A V E   W E   G O O F E D
```

```
C                               ............................
C
90029 CONTINUE
      ANOUPT=TEST+PRNSAM+PUNSAM+PUNCHT+PRINTT+WRITET
      IF(ANOUPT)90035 ,90036 ,90035
90036 PRINT90037
      STOP
C           $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $
C
C
C
C
C                       P R I N T   I N P U T   D A T A   C A R D S
C                       ...........................................
C
C
90035 PRINT90038 ,NOSAMS,SAMSIZ,SCOMAX,(NOCELL(I),I=1,5)
C
      PRINT90039 ,TEST,PRNSAM,PUNSAM,PUNCHT,CONTRL,NOBATS,PRINTT,WRITET,
     1ABSMIN
C
      PRINT90040 ,IBASE(1),IBASE(2)
      IF(WRITET)90041 ,90042 ,90041
90041 CONTINUE
90042 CONTINUE
C
C
C
C
C
```

```
C           -------------------------- P R O G R A M ----------------------
C                                    -------------
C
C
C
C                      B A T C H   L O O P
C                      ...................
C
C
      CASENO=CASENO+1
C
C
C
C
C                                M A S T E R   L O O P
C                                ......................
C
C
C
C
      NCNT=0
      DO90044  NCELL=IBEG,IEND
      NCNT=NCNT+1
      IBASE(1)=KBASE(1)
      IBASE(2)=KBASE(2)
      DO 2002 M=1,200
 2002 COUNT  (M)=0
      ISAMPL =0
      SUMX=0.
      SUMX2=0.
      N=0
      DO90043  BATCH=1,NOBATS
C
      IF(NCELL-1)90045 ,90046 ,90047
90047 IF(NOCELL(NCELL-1))90048,2110   ,90046
C
C
C                    G E N E R A T E   R A N D O M   S A M P L E S
C                    .............................................
C
C
90046 CALLEXPO
      IF(NCELL-1)90045 ,90051 ,90052
90052 CALLDISCRE(TEST,NOSAMS,SAMSIZ,SCOMAX,NOCELL(NCELL-1))
C
C
C
C
C                              P R I N T   S A M P L E S
C                              .........................
C
C
90051 IF(PRNSAM)90054 ,90055 ,90054
C
90054 PRINT90056
      DO90057  K=1,NOSAMS
      PRINT90058 ,(X(K,J),J=1,SAMSIZ)
      PRINT90059 ,(Y(K,J),J=1,SAMSIZ)
90057 CONTINUE
      PRINT90056
```

```
C
C
C
C
C
C                               P U N C H   S A M P L E S
C                               .........................
C
C
90055 IF(PUNSAM)90061 ,90062 ,90061
90061 PUNCH90063 ,NOSAMS,SAMSIZ
      NDUMMY=SAMSIZ/5
C
      DO90064  K=1,NOSAMS
      N1=1
      NI=5
C
      DO90064  I=1,NDUMMY
      PUNCH90065 ,(X(K,J),J=N1,NI),K,N1
      N1=N1+5
      NI=NI+5
90064 CONTINUE
C
      PUNCH90066 ,NOSAMS,SAMSIZ
      DO90067  K=1,NOSAMS
      N1=1
      NI=5
C
      DO90067  I=1,NDUMMY
      PUNCH90068 ,(Y(K,J),J=N1,NI),K,N1
      NI=NI+5
      N1=N1+5
90067 CONTINUE
C
C
C                           S T A T I S T I C A L   T E S T
C                           ...............................
C
C
90062 CONTINUE
      DO90069  K=1,NOSAMS
      CALL TTEST(FMIN,K,NCELL)
90069 CONTINUE
C
C
C
C
C                          E N D  O F   M A S T E R  L O O P
C                          ..................................
C
90049 CONTINUE
C
C
      ISAMPL =ISAMPL +NO  SAMS
      DO 2004 J=1,NO  SAMS
      N=N+1
      SUMX=SUMX+T(J,NCELL)
 2004 SUMX2=SUMX2+T(J,NCELL)*T(J,NCELL)
      DO 2003 J=1,NO  SAMS
      K=(T(J,NCELL)-FMIN)/FINTER
```

```
  2003 COUNT  (K+1)=COUNT  (K+1)+1
       IBASE(1)=JBASE(1)
       IBASE(2)=JBASE(2)
 C
 C                                 E N D   B A T C H   L O O P
 C                                    P R I N T   O U T P U T
 C                                    ......................
       IF(PRINTT)90070 ,90090 ,90070
 90070 PRINT90072
       PRINT90056
       DO90073  K=1,NOSAMS        ,5
       PRINT 2200, T(K,NCELL),T(K+1,NCELL),T(K+2,NCELL),T(K+3,NCELL),T(K+4,N(
      14,NCELL) ,K
  2200 FORMAT(1H ,5F14.8,I10)
 90073 CONTINUE
       PRINT90056
 C
 C
 C                                    P U N C H   O U T P U T
 C                                    ......................
 90090 IF(PUNCHT)90076 ,90077 ,90076
 90076 PUNCH90078 ,NOSAMS,SAMSIZ
       DO90079  K=1,NOSAMS
       PUNCH 2201,T(K,NCELL),K
  2201 FORMAT(F14.8,T73,1HT,I4)
 90079 CONTINUE
 C
 C
 C
 C
```

```
C                                W R I T E   T
C                                .............
C
C
90077 CONTINUE
      IF(WRITET)90081 ,90082 ,90081
90081 CONTINUE
90082 CONTINUE
C                             ...............................
90043 CONTINUE
C
C
      PRINT 2941
 2941 FORMAT(1H1,50X,19HSTATISTICAL SUMMARY)
      PRINT 2943,N
 2943 FORMAT(1H ,54X,I4,6H ITEMS)
      PRINT 2944
 2944 FORMAT(1H0,22X,8H  NUMBER,7X,9HSUM ITEMS,9X,11HSUM SQUARES,12X,5H
     1MEAN,12X,7HSTD DEV)
      AN=N
      AVG=SUMX/AN
      SIGMA=SQRT((SUMX2/AN)-(AVG*AVG))
      PRINT 2922,NCNT,SUMX,SUMX2,AVG,SIGMA
 2922 FORMAT(1H ,26X,I2,4(5X,E15.8))
      FSCAL=ISCAL
      FN=ISAMPL
      PRINT 2030,ISCAL
 2030 FORMAT(1H0,'THE SCALE USED WITH THIS FREQUENCY DISTRIBUTION IS',I4
     1)
      DO 2120 IOUT=1,129
 2120 OUTPUT(IOUT)=BLANK
      NUB=NUMCO+1
      DO 2005 M=1,NUB
      LL=(FLOAT(COUNT(M))/FSCAL)+11.
      IF(LL-11) 2550,2550,2551
 2551 OUTPUT(11)=AST
      IF(LL-119) 2550,2550,2552
 2552 OUTPUT(119)=PER
      GO TO 2553
 2550 OUTPUT(LL)=PER
 2553 MS=M-1
      PRINT 2501,MS,COUNT(M),(OUTPUT(IOUT),IOUT=11,129)
 2501 FORMAT(1H ,2I5    ,119A1)
      OUTPUT(LL)=BLANK
 2005 CONTINUE
      PRINT90084 ,IBASE(1),IBASE(2),CASENO
      PUNCH90085 ,IBASE(1),IBASE(2),CASENO
90044 CONTINUE
C
C
C                        P R I N T   A N D   P U N C H   I B A S E
C                        .........................................
C
C
C
C
C
C
C
 2110  PRINT90084 ,IBASE(1),IBASE(2),CASENO
```

```
      PUNCH90085 ,IBASE(1),IBASE(2),CASENO
C
C
C
C                    R E A D  I N  I N I T I A L  C O N D I T I O N S
C                    ..............................................
C
C
C
C
C
      GO TO 90026
C
C
C
C                                G O O F   C O N D I T I O N S
C                               ( ARE  NOT  TO  OCCUR )
C                                ...........................
C
C
90048 PRINT90086 ,NCELL,NOCELL(NCELL)
C
C
C         / / / / / / / / / / / / / /      / / / / / / / / / / / / /
      STOP
C                                       / / / /
C
90045 PRINT90087 ,NCELL
C
C         / / / / / / / / / / / / / /      / / / / / / / / / / / / /
```

```
      STOP
C                                     ////
C
C
C
C
C           ------------------------- F O R M A T S -------------------
C                                     -------------
C
C
90031 FORMAT('3','R U N    C O M P L E T E D'/' ','**********************
     1*****')
C
90088 FORMAT(15A1)
C
90086 FORMAT('3',,'NOOFCELLS(NCELL) LESS THAN ZERO***NCELL= ',I5,'NOOFCE
     1LLS(NCELL) =',I5)
C
90087 FORMAT('3','$$$$$  NCELL-1 LESS THAN ZERO  NCELL = ',I5)
C
90038 FORMAT('1'/'3','I N I T I A L    C O N D I T I O N S'/' ','*****',
     15X,'NOOFSAMS = ',I5,5X,'SAMPLESIZE = ',I5,5X,'SCOREMAX = ',F7.1/'
     1',10X,'NOOFCELLS = ',(5X,I5))
C
90039 FORMAT(1X,'C O N T R O L     P A R A M E T E R S'/'0','$$$$$',5X,'
     1TEST = ',F5.1,5X,'PRINTSAMS = ',F5.1,5X,'PUNCHSAMS = ',F5.1,5X,'PU
     1NCHT = ',F5.1,5X,'CONTROL = ',F5.1/'0','NUMBER OF BATCHES = ',I5,5
     1X,'PRINT T = ',F5.1,5X,'WRITE T=',1X,F5.1,5X,'ABS T MIN = '1X,F5.1
     1)
C
```

```
90040 FORMAT('0','I BASE (1) FOR X-SAMPLES = ',I15/'0','I BASE (2) FOR Y
     1-SAMPLES = ',I15)
C
90034 FORMAT('SYOU HAVE SUCCESSFULLY LOADED ONTO SCRATCH TAPE AND HAVE C
     1ALLED *RELOAD*')
C
90025 FORMAT('1',///////T20,'* * * * * * * * * * * * * * *  P R O G R A
     1M   S I D O T  * * * * * * * * * * * * * * *'/'0',T50,'R E V   0 4
     1 * 6 5 1 1 1 1')
C
90056 FORMAT('0','. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
     1. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
     1. . . . .')
C
90072 FORMAT('1')
C
90058 FORMAT('0','X SCORES',T20,5F20.8)
C
90059 FORMAT('0','Y SCORES',T20,5F20.8)
C
90074 FORMAT(5F14.8,I10)
C
90084 FORMAT('3','I BASE (1) FOR X = ',I15/'0','I BASE (2) FOR Y = ',I15
     1/'0','COMPUTER SERIAL NUMBER =',6A1,'RAN ON ',8A1,10X,'CASE NUMBER
     1 ',I5)
C
90063 FORMAT(I5,'SAMPLES OF X EACH OF SIZE',I5,T73,'X')
C
90065 FORMAT(5F14.8,T73,'X',I4,I2)
C
```

```
90066 FORMAT(I5,'SAMPLES OF Y EACH OF SIZE',I5,T73,'Y')
C
90068 FORMAT(5F14.8,T73,'Y',I4,I2)
C
90078 FORMAT(I5,'SAMPLES OF T EACH OF SIZE',I5,T73,'T')
C
90080 FORMAT(5F14.8,T73,'T',I4)
C
90085 FORMAT(2I10,'SERIAL NO',6A1,'DAY RUN',8A1,'CASE NO',I5)
C
90027 FORMAT(2I10,F10.4,5I10)
C
90022 FORMAT(5F10.4,I10,F10.4,F10.4)
C
90028 FORMAT(2I10)
C
90037 FORMAT('3','Y O U  H A V E  G O O F E D '/'0','NO OUTPUT HAS BEEN
     1REQUESTED - CORRECT  OUTPUT CONTROL DATA CARD  AND TRY AGAIN - GOO
     1D LUCK ')
C
C
C
C
      END
C
C
C
C       ..........................................................
C
C       ---------------                                    ---------
```

```
      SUBROUTINERRANNO(NOSAMS,SAMSIZ,SCOMAX)
C                               ---------------------------------------
C
C          R E V   0 2 * 6 5 1 1 1 1             G . H .   S N Y D E R , J
C
C          ...............................................................
C
C
C          ------------------ S P E C I F I C A T I O N S ----------------
C                                 ----------------------------
C
C
      INTEGERBASSET,SAMSIZ
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONR(100,20,2),T(100,5),IBASE(2),JBASE(2)
C
C
C          --------- S E T   I N I T I A L   C O N D I T I O N S -----
C                         ------------------------------------------------
C
C
      DO90181  J=1,2
      BASSET=((J-1)*IBASE(J))+((2-J)*IBASE(J))
      CALLSETBAS(BASSET)
C
C
C          ------------------------- P R O G R A M -----------------------
C                                            -------------
```

```
C
C
      DO90182  I=1,SAMSIZ
      DO90182  K=1,NOSAMS
C
      R(K,I,J)=RAND(SCOMAX)
C
90182 CONTINUE
      CALLSAVBAS(JBASE(J))
C
90181 CONTINUE
C
C
      RETURN
C
C
C
      END
C
C
C
C        ..........................................................
C
C        ------------                                  -------------
      SUBROUTINEDISCRE(TEST,NOSAMS,SAMSIZ,SCOMAX,NOCELL)
C        -----                                                    ---
C
C        ..........................................................
C
C
```

```
C           ---------------- S P E C I F I C A T I O N S ---------------
C                                  --------------------------
C
C
      REAL MIN,MAX
      INTEGER BASSET
      INTEGERSAMSIZ,BATCH
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONR(100,20,2)
C
C           --------- S E T   I N I T I A L   C O N D I T I O N S -----
C                       ---------------------------------------------
C
C
      CELWID=SCOMAX/FLOAT(NOCELL)
C
C
C           ------------------------- P R O G R A M -------------------
C                                         ------------
C
C
      IF(TEST)90061 ,90062 ,90061
90061 PRINT90280 ,TEST,NOSAMS,SAMSIZ,SCOMAX,NOCELL,CELWID
90062 DO90281  J=1,2
C
C
      DO90281  K=1,NOSAMS
C
      DO90281  I=1,SAMSIZ
```

```
C
      DO90282  N=1,NOCELL
C
      MAX=(FLOAT(N))*CELWID
      MIN=(FLOAT(N-1))*CELWID
C
      IF(R(K,I,J)-MIN)90283 ,90284 ,90285
90285 IF(R(K,I,J)-MAX)90284 ,90282 ,90282
C
90282 CONTINUE
C
      PRINT90287
      PRINT 12345,R(K,I,J)
12345 FORMAT(F10.3)
      STOP
C
90284 R(K,I,J)=(FLOAT(N-1)+0.5)*CELWID
C
      IF(TEST)90076 ,90281 ,90076
90076 PRINT90058 ,K,I,J,N,MIN,MAX,R(K,I,J)
C
C
90281 CONTINUE
C
      RETURN
C
90283 PRINT90289
      PRINT90058 ,K,I,J,N,MIN,MAX,R(K,I,J)
      STOP
C
```

```
C
C                    ------------------------ F O R M A T S --------------------
C                                                 -------------
C
C
90287 FORMAT('3','N O   N E W R   W A S   F O U N D ')
C
90280 FORMAT('3','..... T E S T   O F   D I S C R E T E'/'0','INPUT',5
     1X,'TEST=',1PE16.8,5X,'NOOFSAMS=',I4,5X,'SAMPLESIZE=',I3/'SCOREMAX
     1= ',1PF16.8,5X,'NOOFC                              ELLS=',I3,5X,'
     1CELLWIDTH=',1PE16.8)
C
90058 FORMAT('0',T5,' K=',I4,' I=',I3,' J=',I2,' N=',I3,'MIN = ',1PE16.8
     1,' MAX=',1PE16.8,' R(K,I,J)=',1PE16.8,)
C
90289 FORMAT(///' ','***** * * *****'/' ',' *   * *   * '/' ',' *   *
     1 *   * '/' ',' *   * *   * '/' ',' *   * *** * '//' ','  ++++
     1+++++++++++++++++++'///)
C
C
C
      END
C
C
C
C           ..........................................................
C
C           ---------------                                ----------------
      SUBROUTINE TTEST(FMIN,K,NCELL)
C                            ------------------------------
```

```
C         R E V  0 1 * 6 5 1 1 1 1           G . H .  S N Y D E R , J
C
C         ............................................................
C
C
C         ---------------- S P E C I F I C A T I O N S ----------------
C                          ---------------------------
C
C
      INTEGERSAMSIZ
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONX(100,20),Y(100,20),T(100,5),IBASE(2),JBASE(2),PRNSAM,PUNSAM
     1,PRINTT,PUNCHT,CONTRL,NOBATS,WRITET,WRISAM,ABSMIN,NOSAMS,SAMSIZ,SC
     1OMAX,NOCELL(5),TEST      ,KBASE(2)
      COMMON MEAN,VAR
C
C
C         --------- S E T   I N I T I A L   C O N D I T I O N S ------
C                   ---------------------------------------------
C
C
      SUMX=0.0
      SUMY=0.0
C
      SUMXSQ=0.0
      SUMYSQ=0.0
C
C
```

```
C                    ------------------------ P R O G R A M ----------------------
C                                             -------------
C
C
      DO90397  I=1,SAMSIZ
      SUMX=SUMX+X(K,I)
      SUMY=SUMY+Y(K,I)
90397 CONTINUE
C
C
      DO90398  I=1,SAMSIZ
      SUMXSQ=SUMXSQ+X(K,I)**2
      SUMYSQ=SUMYSQ+Y(K,I)**2
90398 CONTINUE
      N=SAMSIZ
C
C
      XBAR=SUMX/FLOAT(N)
      YBAR=SUMY/FLOAT(N)
      BARDIF=XBAR-YBAR
      DSUXSQ=SUMXSQ-((SUMX**2)/FLOAT(N))
      DSUYSQ=SUMYSQ-((SUMY**2)/FLOAT(N))
      DISUSQ=DSUXSQ+DSUYSQ
C
      IF(DISUSQ) 1,1,2
  1   T(K,NCELL)=0
      GO TO 90401
  2   CONTINUE
      NUMTIM=NUMTIM+1
      T(K,NCELL)=BARDIF/SQRT(DISUSQ*(1.0/((FLOAT(N)-1.0)*FLOAT(N))))
      IF(ABS(T(K,NCELL))-ABSMIN)90401 ,90401 ,90402
90402 CONTINUE
      T(K,NCELL)=ABSMIN*(T(K,NCELL)/ABS(T(K,NCELL)))
90401 CONTINUE
C
      IF(TEST)90054 ,90055 ,90054
C
C
90054 PRINT90403
      PRINT90056
      PRINT90404
C
      DO90405  I=1,SAMSIZ
      PRINT90280 ,I,X(K,I),Y(K,I),K
90405 CONTINUE
      PRINT90058 ,SUMX,SUMY,SUMXSQ,SUMYSQ,SAMSIZ
      PRINT90059 ,XBAR,YBAR,DSUXSQ,DSUYSQ
      PRINT90074 ,T(K,NCELL)
      PRINT90406
      PRINT90056
      PRINT90403
C
C
90055 RETURN
C
C
C               ----------------------- F O R M A T S ---------------------
C                                       ------------
C
C
```

```
90404 FORMAT(///' ',T40,'***  **** ****  * *   *     ***** **** **** ***
     1**'/' ',T40,'*  * *    *     * **  *       *   *    *      *  '/'
     1',T40,'***  ***  * *** * * * *       *   ***  ****   *  '/' ',T40,
     1'*  * *    *  *  * *  **       *   *       *   *  '/' ',T40,'***
     1**** ****  * *   *       *   **** ****   *  '//' ',T40,'..........
     1.......................................'///)
C
90406 FORMAT(///' ',T40,'...............................................
     1..'//' ',T44,'**** *   * ***      ***** **** **** *****'/' ',T44,'
     1*    **  * *  *       *   *    *      *  '/' ',T44,'***  * * * *
     1*       *   ***  ****   *  '/' ',T44,'*    *  ** *  *       *   *
     1      *   *  '/' ',T44,'**** *   * ***        *   **** ****   *  '
     1///)
C
90056 FORMAT('0','. . . . . . . . . . . . . . . . . . . . . . . . . . .
     1. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
     1. . . . .')
C
C
90280 FORMAT(///'0','I = ',I5,T15,'X = ',F15.7,T40,'Y = ',F15.7,T120,'K
     1= ',I3)
C
90058 FORMAT('0',T12,'SUMX = ',F15.7,T37,'SUMY = ',F15.7,T62,'SUMXSQR =
     1',E16.8,T92,'SUMYSQR = ',E16.8,T120,'N = ',I5)
C
90059 FORMAT('0',T12,'XBAR = ',F15.7,T37,'YBAR = ',F15.7,T64,'DIFSUMXSQR
     1 = ',E16.8,T100,'DIFSUMYSQR = ',E16.8)
C
90074 FORMAT('0','T = ',E16.8,T40,'E N D   P R I N T - RETURN'////)
C
```

```
90403 FORMAT(///)
C
C
C
      END
      SUBROUTINE EXPO
      INTEGERSAMSIZ,BATCH
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMON          R(100,20,2),T(100,5),IBASE(2),JBASE(2),PRNSAM,PUNSAM
     1,PRINTT,PUNCHT,CONTRL,NOBATS,WRITET,WRISAM,ABSMIN,NOSAMS,SAMSIZ,SC
     1OMAX,NOCELL(5),TEST     ,KBASE(2)
      COMMON MEAN,VAR
      SCOMAX=1.0
      IBASE(1)=1009666569
      IBASE(2)=1291401631
      MEAN= 0.500
      RM=MEAN*2.0
      AM=MEAN
      DO 1 J=1,2
      BASSE =((J-1)*IBASE(J))+((2-J)*IBASE(J))
      CALL SETBAS(BASSE)
      DO 2 I=1,SAMSIZ
      DO 2 K=1,NOSAMS
90001 R     (K,I,J)=(-1.0)*AM*ALOG(RAND(SCOMAX))
      IF(R(K,I,J)-RM)4,4,90001
4     CONTINUE
C
C
```

```
  2   CONTINUE
  1   CONTINUE
      RETURN
      END
      SUBROUTINE RNDSCO
      INTEGERSAMSIZ,BATCH
      INTEGERTIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGERTOTTIE
      INTEGERTTIIN
      REALIUNRNK
      INTEGER BASSET
      REAL MEAN
      REAL NORM
      DIMENSIONNOOFCL(5),TOTTIE(5)
C
      COMMON IRNSTR,POS(40)
      COMMON         R(100,20,2),T(100,5),IBASE(2),JBASE(2),PRNSAM,PUNSAM
     1,PRINTT,PUNCHT,CONTRL,NOBATS,WRITET,WRISAM,ABSMIN,NOSAMS,SAMSIZ,SC
     1OMAX,NOCELL(5),TEST      ,KBASE(2)
      COMMON MEAN,VAR
      DO 1 J=1,2
      BASSET=((J-1)*IBASE(J))+((2-J)*IBASE(J))
      CALLSETBAS(BASSET)
      DO 5 I=1,SAMSIZ
      DO 2 K=1,NOSAMS
  4   N = RAND(2.) + 1.                                                 OS/
      ETA = SQRT(ALOG(1./RAND(.5)**2))                                  OS/
      X = (ETA - (2.515517 + ETA*(.802853 + .010328 * ETA))
     1/ (1. + ETA*(1.432788 + ETA*(.189269 + .001308*ETA)))
     2)*(-1.)**N
```

```
      R(K,I,J)=MEAN+VAR*X
      IF(ABS(R(K,I,J)).GT.5)GO TO 4
 2    CONTINUE
5     CONTINUE
      CALL SAVBAS(JBASE(J))
 1    CONTINUE
      RETURN
      END
/*
//DATA.INPUT DD *
       100        10    1.            50        20        10        05
       0.0      0.0        0.0       0.0       1.0        50       0.0
10096665691291401631
         1         6
        05    +5.5        -5.5        50       100
        .5
       8.0

/*
```

```
//            'U3655,T=2000,R=6000,UTEST','DEAGMAN G K',MSGLEVEL=1
// EXEC FHCLG,PARM.SOURCE='MAP,OPT=2'
//SOURCE.INPUT DD *
C
C
C         ..............................................................
C
C         * * * * * * * * *  P R O G R A M   S I D O U  * * * * * * * * *
C                            * * * * * * * * * * * * * *
C
C         R E V 0 2 * 6 6 0 5 0 6                    G H  S N Y D E R   J R
C
C         1-INCLUDE NUMBER OF BATCHES
C         2-CORRECT STATEMENT  OUT 2
C         3-ADD   PRINT U   CONTROL
C         4-ADD  LOGIC TO STOP PROGRAM IF NO OUTPUT IS REQUESTED
C R E V 0 3 UPDATED TO USE SUBROUTINES FOR FORM UNION,ORDER UNION ,GET U
C66 05 12  PREPARATION FOR HANDELING TIES
C
C         ..............................................................
C
C
C
C
C                                 D A T A   C A R D S
C                                 ===================
C
C
CNOTE            FIRST DATA CARD MUST BE VALUE OF TEST DOC
C           TEST DOC = 1.0 GIVES TEST OF DOCUMENT SUBROUTINE
```

```
C             TEST DOC =0.0 NO TEST
C                     NEXT FOUR DATA CARDS MUST BE DOCUMENTARY CARDS FOR
C                     SUBROUTINE DOCUMENT
C        THE LAST DATA CARD MUST BE A BLANK
C
C
C             ----------------- S P E C I F I C A T I O N S --------------
C                                  ---------------------------
C
C
C
      LOGICAL*1OUTPUT(129)
      LOGICAL*1PER/'.'/, BLANK/' '/, AST/'*'/
      INTEGER COUNT  (200)
      INTEGERCASENO
      INTEGER BASSET
      REAL MEAN
      INTEGERSAMSIZ,BATCH
      INTEGERTIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGERTOTTIE
      INTEGERTTIIN
      REALIUNRNK
C
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      DIMENSION KBASE(2)
      DIMENSIONNOOFCL(5),TOTTIE(5)
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
```

```
C
C
C          ---------- S E T   I N I T I A L   C O N D I T I O N S ----
C                        ----------------------------------------------
C
C
      CALL SETBTF
      IRNSTR=0
      PRINT90042
      CALLORDUNI(1,1.0,1)
      DO90001  I=1,5
      TOTTIE(I)=0
      TTIIN (I)=0
      TIINDE(I)=0
90001 CONTINUE
C
C
      CASENO=0
C
C
C                         R E A D   I N P U T   D A T A
C                         ..............................
C
C
90002 READ90058 ,NOSAMS,SAMSIZ,SCOMAX,(NOOFCL(I),I=1,5)
      IF(NOSAMS) 3010,3010,3011
 3010 STOP
 3011 READ90059 ,TEST,PRNSAM,PUNSAM,PNCHU ,CNTROL,NOBATS,PRINTU,WRITEU
C
C
```

```
      READ90060 ,IBASE(1),IBASE(2),IBEG,IEND
      KBASE(1)=IBASE(1)
      KBASE(2)=IBASE(2)
 2111 CONTINUE
      READ 2100,N,FMAX,FMIN,NUMCO    ,ISCAL
 2100 FORMAT(1(8X,I2),2F10.0,2(7X,I3))
      READ 3001,MEAN,VAR
 3001 FORMAT(2F10.3)
      VAR=SQRT(VAR)
      Z=FMAX-FMIN
      FINTER   =Z/FLOAT(NUMCO   )
      PRINT 2000,FMIN,FMAX,FINTER
 2000 FORMAT('1THE MINIMUM SAMPLE IS ',1PE14.7,'.  THE MAXIMUM SAMPLE IS
     1 ',E14.7,'.  THE STEP USED IS ',E14.7)
      PRINT 2099
 2099 FORMAT('OTHE FOLLOWING ARE THE INTERVALS USED AND THEIR RANGES.'/'
     10',T27,'INTERVAL',T45,'LOWER LIMIT',T66,'UPPER LIMIT')
      II=NUMCO   -1
      RANGE=FMIN
      DO 2001 M=1,II
      XX=RANGE
      YY=XX+FINTER
      RANGE=YY
 2001 PRINT 3000,M,XX,YY
 3000 FORMAT('O',T29,I4,T44,1PE14.7,T65,E14.7)
      M=NUMCO
      XX=FMAX
      PRINT 3000,M,YY,XX
C
C
```

```
C
C
C                       I S   R U N   C O M P E T E D
C                       ............................
C
C
      IF(CNTROL)90003 ,90004 ,90003
90004 PRINT90036
      IF(WRITEU)90005 ,90006 ,90005
90005 CONTINUE
90006 CONTINUE
      PRINT90062
      STOP
C           $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $
C
C
C
C                             H A V E   W E   G O O F E D
C                             ..........................
C
90003 CONTINUE
      ANOUPT=TEST+PRNSAM+PUNSAM+PNCHU +PRINTU+WRITEU
      IF(ANOUPT)90007 ,90008 ,90007
90008 PRINT90061
      STOP
C           $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $
C
C
C
C
```

```
C                          P R I N T   I N P U T   D A T A   C A R D S
C                          ..........................................
C
C
90007 PRINT90037 ,NOSAMS,SAMSIZ,SCOMAX,(NOOFCL(I),I=1,5)
C
      PRINT90038 ,TEST,PRNSAM,PUNSAM,PNCHU ,CNTROL,NOBATS,PRINTU,WRITEU
     1
C
      PRINT90039 ,IBASE(1),IBASE(2)
      IF(WRITEU)90009 ,90010 ,90009
90009 CONTINUE
90010 CONTINUE
C
C
C
C
C           ----------------------- P R O G R A M ----------------------
C                                   ------------
C
C
C
C
C                            B A T C H   L O O P
C                            ...................
C
C
      CASENO=CASENO+1
C                                      M A S T E R   L O O P
C                                      .....................
```

```
C
C
C
C
      NCNT=0
      DO90026  NCELL=IBEG,IEND
      NCNT=NCNT+1
      IBASE(1)=KBASE(1)
      IBASE(2)=KBASE(2)
      DO 2002 M=1,200
 2002 COUNT  (M)=0
      DO 2070 IOUT=1,129
 2070 OUTPUT(IOUT)= BLANK
      ISAMPL =0
      SUMX=0.
      SUMX2=0.
      N=0
      DO90035  BATCH=1,NOBATS
C
C
C
C
      IF(NCELL-1)90011 ,90012 ,90013
90013 IF(NOOFCL(NCELL-1))90014 , 2110 ,90012
C
C
C                     G E N E R A T E   R A N D O M   S A M P L E S
C                     ..............................................
C
C
```

```
90012 CALLRNDSCO(SAMSIZ)
      IF(NCELL-1)90011 ,90016 ,90017
90017 CALLDISCRT(TEST,SAMSIZ,SCOMAX,NOOFCL(NCELL-1))
C
C
C
C
C                               P R I N T   S A M P L E S
C                               .........................
C
C
90016 IF(PRNSAM)90018 ,90019 ,90018
C
90018 PRINT90043
      DO90020  K=1,NOSAMS
      PRINT 90045,(R(K,J,1),J=1,SAMSIZ)
      PRINT 90046,(R(K,J,2),J=1,SAMSIZ)
90020 CONTINUE
      PRINT90043
C
C
C
C
C                               P U N C H   S A M P L E S
C                               .........................
C
C
90019 IF(PUNSAM)90021 ,90022 ,90021
90021 PUNCH90051 ,NOSAMS,SAMSIZ
      NDUMMY=SAMSIZ/5
```

```
C
      DO90023  K=1,NOSAMS
      N1=1
      NI=5
C
      DO90023  I=1,NDUMMY
      PUNCH 90052,(R(K,J,1),J=N1,NI),K,N1
      N1=N1+5
      NI=NI+5
90023 CONTINUE
C
      PUNCH90053 ,NOSAMS,SAMSIZ
      DO90024  K=1,NOSAMS
      N1=1
      NI=5
C
      DO90024  I=1,NDUMMY
      PUNCH 90054,(R(K,J,2),J=N1,NI),K,N1
      NI=NI+5
      N1=N1+5
90024 CONTINUE
C
C
C
C
C                          S T A T I S T I C A L   T E S T
C                          ................................
C
C
90022 DO90025  K=1,NOSAMS
```

```
      CALL UTEST(TEST,SAMSIZ,K,NCELL,FMIN)
      TOTTIE(NCELL)=TOTTIE(NCELL)+NOOFTI(NCELL)
      TTIIN (NCELL)=TTIIN (NCELL)+TIINDE(NCELL)
90025 CONTINUE
C
C
C
90015 CONTINUE
      ISAMPL =ISAMPL +NO  SAMS
      DO 2004 J=1,NO  SAMS
      N=N+1
      SUMX=SUMX+U(J,NCELL)
 2004 SUMX2=SUMX2+U(J,NCELL)*U(J,NCELL)
      DO 2003 J=1,NO  SAMS
      K=(U(J,NCELL)-FMIN)/FINTER
 2003 COUNT  (K+1)=COUNT  (K+1)+1
      IBASE(1)=JBASE(1)
      IBASE(2)=JBASE(2)
C
C                                 E N D   B A T C H   L O O P
C                                 ..............................
C
C                              E N D  O F  M A S T E R  L O O P
C                              ..................................
C
C                                      P R I N T  O U T P U T
C                                      ......................
      IF(PRINTU)90027 ,90028 ,90027
90027 PRINT90044
      PRINT90043
      DO90029  K=1,NOSAMS       ,5
      PRINT 90047,U(K,NCELL),U(K+1,NCELL),U(K+2,NCELL),U(K+3,NCELL),U(K+
     14,NCELL),K
90029 CONTINUE
      PRINT90043
C
C
C                                   P U N C H   O U T P U T
C                                   .......................
C
C
90028 IF(PNCHU )90030 ,90031 ,90030
90030 PUNCH90055 ,NOSAMS,SAMSIZ
      DO90032  K=1,NOSAMS
      PUNCH 90056,U(K,NCELL),K
90032 CONTINUE
C
C
90031 CONTINUE
C
      IF(WRITEU)90033 ,90034 ,90033
90033 CONTINUE
90034 CONTINUE
C
90035 CONTINUE
C
C
      PRINT 2941
 2941 FORMAT(1H1,50X,29HSTATISTICAL SUMMARY,U-NOTIES )
      PRINT 2943,N
 2943 FORMAT(1H ,54X,I4,6H ITEMS)
      PRINT 2944
 2944 FORMAT(1H0,22X,8H  NUMBER,7X,9HSUM ITEMS,9X,11HSUM SQUARES,12X,5H
     1MEAN,12X,7HSTD DEV)
      AN=N
      AVG=SUMX/AN
      SIGMA=SQRT((SUMX2/AN)-(AVG*AVG))
      PRINT 2922,NCNT,SUMX,SUMX2,AVG,SIGMA
 2922 FORMAT(1H ,26X,I2,4(5X,E15.8))
      FSCAL=ISCAL
      FN=ISAMPL
      PRINT 2030,ISCAL
 2030 FORMAT(1H0,'THE SCALE USED WITH THIS FREQUENCY DISTRIBUTION IS',I4
     1)
      NUB=NUMCO+1
      DO 2005 M=1,NUB
      LL=(FLOAT(COUNT(M))/FSCAL)+11.
      IF(LL-11) 2550,2550,2551
 2551 OUTPUT(11)=AST
      IF(LL-119) 2550,2550,2552
 2552 OUTPUT(119)=PER
      GO TO 2553
 2550 OUTPUT(LL)=PER
 2553 MS=M-1
      PRINT 2501,MS,COUNT(M),(OUTPUT(IOUT),IOUT=11,129)
 2501 FORMAT(1H ,2I5    ,119A1)
      OUTPUT(LL)=BLANK
 2005 CONTINUE
90026 CONTINUE
C
```

```
C                           E N D   B A T C H   L O O P
C                           ..............................
C
C
C
      PRINT90049 ,(TOTTIE(I),I=1,5)
      PRINT90050 ,(TTIIN (I),I=1,5)
C
C                     P R I N T   A N D   P U N C H   I B A S E
C                     .........................................
C
C
C
C
 2110 CONTINUE
      PUNCH90057 ,IBASE(1),IBASE(2)
      PRINT90048 ,IBASE(1),IBASE(2)
C
C
C
C                  R E A D   I N   I N I T I A L   C O N D I T I O N S
C                  .................................................
C
C
C
C
      GO TO 90002
C
C
C
```

```
C                                  G O O F   C O N D I T I O N S
C                                  ( ARE  NOT  TO  OCCUR )
C                                  .............................
C
C
90014 PRINT90040 ,NCELL,NOOFCL(NCELL)
      STOP
C               $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $
C
C
90011 PRINT90041 ,NCELL
      STOP
C               $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $
C
C
C
C
C               ------------------------ F O R M A T S ----------------------
C                                        -------------
C
C
C
90036 FORMAT('3','R U N    C O M P L E T E D'/' ','**********************
     1*****')
C
90037 FORMAT('1'/'3','I N I T I A L    C O N D I T I O N S'/' ','*****',
     15X,'NOOFSAMS = ',I5,5X,'SAMPLESIZE = ',I5,5X,'SCOREMAX = ',F7.1/'
     1',10X,'NOOFCELLS = ',(5X,I5))
C
90038 FORMAT(1X,'C O N T R O L     P A R A M E T E R S'/'0','$$$$$',5X,'
     1TEST = ',F5.1,5X,'PRINTSAMS = ',F5.1,5X,'PUNCHSAMS = ',F5.1,5X,'PU
     1NCHT = ',F5.1,5X,'CONTROL = ',F5.1/'0','NUMBER OF BATCHES = ',I5,5
     1X,'PRINT U = ',F5.1,5X,'WRITE U=',1X,F5.1)
C
90039 FORMAT('0','I BASE (1) FOR X-SAMPLES = ',I15/'0','I BASE (2) FOR Y
     1-SAMPLES = ',I15)
C
90040 FORMAT('3',,'NOOFCELLS(NCELL) LESS THAN ZERO***NCELL= ',I5,'NOOFCE
     1LLS(NCELL) =',I5)
C
90041 FORMAT('3','$$$$$  NCELL-1 LESS THAN ZERO  NCELL = ',I5)
C
90042 FORMAT('1',///////T20,'* * * * * * * * * * * * * * *  P R O G R A
     1M   S I D O U  * * * * * * * * * * * * * * *'/'0',T50,'R E V   0 2
     1 * 6 6 0 5 0 6'/'0','ADD NO. OF BATCHES ,ADD PRINT U  CONTROL   AN
     1D ALSO CORRECT STATEMENT-OUT 2 '/'0','ADD LOGIC TO STOP PROGRAM IF
     1 NO OUTPUT REQUESTED '/'0','R E V  0 3  -  66 05 12'/'0','REVISED
     1TO INCLUDE SUBROUTINES*FORM UNION,ORDER UNION,GET U   IN PREPARATI
     1ON FOR HANDELING TIES ')
C
90043 FORMAT('0','. . . . . . . . . . . . . . . . . . . . . . . . . . .
     1. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
     1. . . . .')
C
90044 FORMAT('1')
C
90045 FORMAT('0','X SCORES',T20,5F20.8)
C
90046 FORMAT('0','Y SCORES',T20,5F20.8)
C
```

```
90047 FORMAT(1H ,5F14.8,I10)
C
90048 FORMAT('3','I BASE (1) FOR X = ',I15/'0','I BASE (2) FOR Y = ',I15
     1/'0','COMPUTER SERIAL NUMBER =',6A1,'RAN ON ',8A1,10X,'CASE NUMBER
     1 ',I5)
90049 FORMAT('0''TOTAL TIES = ',5(3X,I10))
90050 FORMAT('0''TTIE INDEED = ',5(3X,I10))
C
90051 FORMAT(I5,'SAMPLES OF X EACH OF SIZE',I5,T73,'X')
C
90052 FORMAT(5F14.8,T73,'X',I4,I2)
C
90053 FORMAT(I5,'SAMPLES OF Y EACH OF SIZE',I5,T73,'Y')
C
90054 FORMAT(5F14.8,T73,'Y',I4,I2)
C
90055 FORMAT(I5,'SAMPLES OF U EACH OF SIZE',I5,T73,'U')
C
90056 FORMAT( F14.8,T73,'U',I4)
C
90057 FORMAT(2I10,'SERIAL NO',6A1,'DAY RUN',8A1,'CASE NO',I5)
C
90058 FORMAT(2I10,F10.4,5I10)
C
90059 FORMAT(5F10.4,I10,F10.4,F10.4)
C
90060 FORMAT(2I10)
C
90061 FORMAT('3','Y O U  H A V E  G O O F E D '/'0','NO OUTPUT HAS BEEN
     1REQUESTED - CORRECT  OUTPUT CONTROL DATA CARD  AND TRY AGAIN - GOO
     1D LUCK ')
C
90062 FORMAT('SYOU HAVE SUCCESSFULLY LOADED ONTO SCRATCH TAPE AND HAVE C
     1ALLED *RELOAD*')
C
C
C
      END
      FUNCTIONITIME(I)
      ITIME=0
      RETURN
      END
C
C
C
C         .................................................................
C
C         ---------------                                         ---------
      SUBROUTINERECRAN(SAMSIZ,SCOMAX)
C                         -----------------------------------
C
C         R E V   0 2 * 6 5 1 1 1 1             G . H .   S N Y D E R , J
C
C         .................................................................
C
C
C         ----------------- S P E C I F I C A T I O N S -----------------
C                            --------------------------
C
C
```

```
      INTEGERBASSET,SAMSIZ
      INTEGER TIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGER TTIIN
      REAL  IUNRNK
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
C
C
C          ---------- S E T   I N I T I A L   C O N D I T I O N S ------
C                        ------------------------------------------------
C
C
      DO90064  J=1,2
      BASSET=((J-1)*IBASE(J))+((2-J)*IBASE(J))
      CALLSETBAS(BASSET)
C
C
C                                              --------------
C
C
      DO90063  I=1,SAMSIZ
      DO90063  K=1,NOSAMS
C
      R(K,I,J)=RAND(SCOMAX)
C
90063 CONTINUE
```

```
      CALLSAVBAS(JBASE(J))
C
90064 CONTINUE
C
C
      RETURN
C
C
C
      END
C
C
C
C          ...........................................................
C
C          -------------                                  --------------
      SUBROUTINEDISCRT(TEST,SAMSIZ,SCOMAX,NOOFCL)
C          -----                                                     ---
C
C          ...........................................................
C
C
C          ----------------- S P E C I F I C A T I O N S ----------------
C                            ---------------------------
C
C
      INTEGERSAMSIZ
      REALMIN,MAX
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMONR(100,20,2)
C
C
C         ---------S E T   I N I T I A L   C O N D I T I O N S-----
C                       ---------------------------------------
C
C
      CELWID=SCOMAX/FLOAT(NOOFCL)
C
C
C         ---------------------- P R O G R A M ----------------------
C                                -----------
C
C
      IF(TEST)90065 ,90066 ,90065
90065 PRINT90074 ,TEST,NOSAMS,SAMSIZ,SCOMAX,NOOFCL,CELWID
90066 DO90072  J=1,2
C
C
      DO90072  K=1,NOSAMS
C
      DO90072  I=1,SAMSIZ
C
      DO90070  N=1,NOOFCL
C
      MIN=(FLOAT(N-1))*CELWID
      MAX=(FLOAT(N))*CELWID
C
```

```
      IF(R(K,I,J)-MIN)90067 ,90068 ,90069
90069 IF(R(K,I,J)-MAX)90068 ,90070 ,90070
C
90070 CONTINUE
C
      PRINT90073
      STOP
C
90068 R(K,I,J)=(FLOAT(N-1)+0.5)*CELWID
C
      IF(TEST)90071 ,90072 ,90071
90071 PRINT90075 ,K,I,J,N,MIN,MAX,R(K,I,J)
C
C
90072 CONTINUE
C
      RETURN
C
90067 PRINT90076
      PRINT90075 ,K,I,J,N,MIN,MAX,R(K,I,J)
      STOP
C
C
C          ------------------------ F O R M A T S ----------------------
C                                   -------------
C
C
90073 FORMAT('3','N O   N E W R   W A S   F O U N D ')
C
90074 FORMAT('3','..... T E S T   O F   D I S C R E T E'/'0','INPUT',5
      1X;'TEST=';1PE16.8,5X;'NOOFSAMS=',I4,5X,'SAMPLESIZE=',I3/'SCOREMAX
      1= ';1PE16.8;5X;'NOOFC                                    ELLS=',I3,5X,'
      1CELLWIDTH=';1PE16.8)
C
90075 FORMAT('0',T5,' K=',I4,' I=',I3,' J=',I2,' N=',I3,'MIN = ',1PE16.8
      1,' MAX=',1PE16.8,' R(K,I,J)=',1PE16.8,)
C
90076 FORMAT(///' ','***** * * *****'/' ',' *   * *   *  '/' ','  *   *
      1 *   *  '/' ',' *   * *   *  '/' ','  *   * *** *  '//' ',' ++++
      1++++++++++++++++++++'///)
C
C
C
      END
C
C
C      ..........................................................
C
C
C      -----------------                                   -------------
      SUBROUTINE UTEST(TEST,SAMSIZ,K,NCELL,FMIN)
C                          ----------------------------------
C
CR E V 0 1  REVISED TO INCORPORATE FORM UNION,ORDER UNION,GET U
C
C      ..........................................................
C
C
C
C
```

```
C              ----------------- S P E C I F I C A T I O N S ---------------
C                                  ---------------------------
C.
C
      INTEGER TIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGER TTIIN
      INTEGERSAMSIZ,TIMRUN,OULOMX
      REALIUNRNK
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
C
C
C
C              --------- S E T   I N I T I A L   C O N D I T I O N S -----
C                        ----------------------------------------------
C
C
C
      INDEX=2*SAMSIZ
      INTGER=0
      TIMRUN=ITIME(INTGER)
C
C
C              ------------------------- P R O G R A M -------------------
C                                         -------------
```

```
C
C
C
      IF(TEST)90077 ,90078 ,90077
C
90077 PRINT90083
      PRINT90085 ,TIMRUN,TEST,SAMSIZ,K,NCELL
C
90078 CONTINUE
C
      CALLFRMUNI(SAMSIZ,K)
C
      IF(TEST)90079 ,90080 ,90079
C
90079 CONTINUE
      J=1
      PRINT90086 ,(R(K,I,J),I=1,SAMSIZ)
C
      J=2
      PRINT90087 ,(R(K,I,J),I=1,SAMSIZ)
      PRINT90088 ,(XUNIY (L),L=1,INDEX)
      PRINT90089 ,(IUNRNK(L),L=1,INDEX)
90080 CONTINUE
C
C
      CALLORDUNI(INDEX,0.0,NCELL)
      CALLGETU(SAMSIZ,INDEX,NCELL,K,TEST)
C
C
      IF(TEST)90081 ,90082 ,90081
```

```
90081 TIMRUN=ITIME(INTGER)
      PRINT90088 ,(XUNIY (L),L=1,INDEX)
      PRINT90089 ,(IUNRNK(L),L=1,INDEX)
      PRINT90084
C
C
90082 CONTINUE
      RETURN
C
C
C
C
C
C
C                 ------------------------ F O R M A T S ----------------------
C                                          -------------
C
C
C
90083 FORMAT(///' ',T40,'***  **** ****  * *   *     ***** **** **** ***
     1**'/' ',T40,'*  * *    *     * **  *       *   *    *      *  '/'
     1',T40,'***  ***  * *** * * * *        *   ***  ****   *  '/' ',T40,
     1'*  * *    *  *  * *  **       *   *       *   *  '/' ',T40,'***
     1**** ****  * *   *       *   **** ****   *  '//' ',T40,'..........
     1......................................'///)
C
90084 FORMAT(///' ',T40,'.........................................................
     1..'//' ',T44,'**** *   * ***      ***** **** **** *****'/' ',T44,'
     1*   ** * *  *       *   *   *      *  '/' ',T44,'***  * * * *
     1*      *  ***  ****   *  '/' ',T44,'*    *  ** *  *       *   *
     1      *   *  '/' ',T44,'**** *   * ***        *   **** ****  *  '
     1///)
C
90085 FORMAT('0','S U B R O U T I N E   U T E S T',T100,'COMPUTER TIME =
     1 ',I5/'0',6X,'C A L L E D   V A L U E S',5X,'TEST = ',1PE16.8,5X,'
     1SAMPLESIZE = ',I5,5X,'SAMPLENO. = ',I5,5X,'NCELL = ',I3)
C
C
90086 FORMAT('0',T51,'X-SAMPLES',(/1X,5F20.8))
90087 FORMAT('0',T51,'Y-SAMPLES',(/1X,5F20.8))
90088 FORMAT('0',T60,'WORKINGR(L)',(/1X,5F20.8))
90089 FORMAT('0',T60,'WORKINGU(L)',(/1X,5F20.8))
C
C
C
C
      END
      SUBROUTINEFRMUNI(SAMSIZ,K)
      INTEGERSAMSIZ
      INTEGER TIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGER TTIIN
      REALIUNRNK
      INTEGER BASSET
      REAL MEAN
C
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
      DO90090  I=1,SAMSIZ
      DO90090  J=1,2
      L=I+(J-1)*SAMSIZ
      XUNIY (L)=R(K,I,J)
      IUNRNK(L)=FLOAT(J)
90090 CONTINUE
C
      RETURN
      END
      SUBROUTINEORDUNI(INDEX,HOWTIE,NCELL)
      INTEGEROULOMX
      INTEGER TIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGER TTIIN
      INTEGERTIEFLG
      REAL  IUNRNK
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
      DO90091  I=1,20
      BEGNTI(I)=0
      ENDTIE(I)=0
90091 CONTINUE
      DO90092  I=1,5
      NOOFTI(I)=0
90092 CONTINUE
      IF(HOWTIE)90093 ,90094 ,90093
```

```
90093 PRINT90095
90095 FORMAT('0',10(1H+)/10X,' TIES ARE HANDELED BY TAKING X-SCORES BEFO
     1RE Y-SCORES')
90094 CONTINUE
      TIEGRU=0
      TIEFLG=0
      OULOMX=INDEX-1
C
      DO90102  LOUT=1,OULOMX
      INLOMX=INDEX-LOUT
C
      DO90100  LIN=1,INLOMX
      IF(XUNIY (LIN)-XUNIY (LIN+1))90096 ,90097 ,90098
C
C
90096 CONTINUE
90099 GO TO 90100
90097 CONTINUE
90101 GO TO 90100
90098 CONTINUE
C
C
      DUMMYR=XUNIY (LIN)
      XUNIY (LIN)=XUNIY (LIN+1)
      XUNIY (LIN+1)=DUMMYR
C
      DUMMYU=IUNRNK(LIN)
      IUNRNK(LIN)=IUNRNK(LIN+1)
      IUNRNK(LIN+1)=DUMMYU
      GO TO 90100
```

```
C
90100 CONTINUE
C
C
90102 CONTINUE
      DO 1 I=1,INDEX
    1 POS(I)=I
      IN=INDEX-1
      DO90108  LTI=1,IN
      IF(XUNIY (LTI)-XUNIY (LTI+1))90103 ,90104 ,90105
90103 IF(TIEFLG)90106 ,90107 ,90106
90107 GO TO 90108
90106 TIEFLG=0
      ENDTIE(TIEGRU)=LTI
      GO TO 90108
90104 IF(TIEFLG)90109 ,90110 ,90109
90110 TIEFLG=1
      TIEGRU=TIEGRU+1
      BEGNTI(TIEGRU)=LTI
      NOOFTI(NCELL)=NOOFTI(NCELL)+2
      GO TO 90108
90109 NOOFTI(NCELL)=NOOFTI(NCELL)+1
      GO TO 90108
90108 CONTINUE
      IF(TIEFLG)90111 ,90112 ,90111
90111 ENDTIE(TIEGRU)=INDEX
90112 IF(TIEGRU)90113 ,90114 ,90113
90113 NTIRGR=TIEGRU
      CALLTIEIND(NCELL,NTIRGR)
90114 RETURN
```

```
90105 PRINT90115
90115 FORMAT('0',10(1H+)/10X,'YOU CANNOT GET THIS')
      STOP
      END
      SUBROUTINEGETU(SAMSIZ,INDEX,NCELL,K,TEST)
      INTEGERSAMSIZ,TIMRUN
      INTEGER TTINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGER TTIIN
      REAL  IUNRNK
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TTINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
C
C
      INTGER=0
      TIMRUN=ITIME(INTGER)
      RANKX=0.0
      RANKY=0.0
C
      DO90116  L=1,INDEX
      RANKX=RANKX+FLOAT(L)*(2.0-IUNRNK(L))
      RANKY=RANKY+FLOAT(L)*(IUNRNK(L)-1.0)
90116 CONTINUE
C
C
      NOTHNG=((SAMSIZ*((3*SAMSIZ)+1))/2)
```

```
C
      UX=FLOAT(NOTHNG)-RANKX
      UY=FLOAT(NOTHNG)-RANKY
      U(K,NCELL)=AMIN1(UX,UY)
      IF(TEST)90117 ,90118 ,90117
90117 PRINT90119 ,NOTHNG,RANKX,RANKY,U(K,NCELL),TIMRUN
90118 CONTINUE
C
C
90119 FORMAT('0','NOTHING = ',I10,5X,'RANKX = ',1PE16.8,5X,'RANKY = ',1P
     1E16.8,5X,'U(K,NU) = ',1PE16.8,'TIMERUN = ',I10)
C
      RETURN
      END
      SUBROUTINETIEIND(NCELL,NTIGRU)
      INTEGER TIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGER TTIIN
      REAL  IUNRNK
      REAL MEAN
      INTEGER BASSET
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5),NOOFTI(5)
      COMMON MEAN,VAR
      IF(NOOFTI(NCELL)) 90123,90124,90123
90124 RETURN
90123 CONTINUE
      BEGIN=0
      END=0
      DO90135  I=1,5
      TIINDE(I)=0
90135 CONTINUE
      DO90134  TIEGRU=1,NTIGRU
      NOXTIE=0
      NOYTIE=0
      NTIES=0
      BEGIN=BEGNTI(TIEGRU)
      END=ENDTIE(TIEGRU)
      DO90128  I=BEGIN,END
      IF(IUNRNK(I)-1.0) 90125,90126,90125
90125 NOYTIE=NOYTIE+1
      GO TO 90127
90126 NOXTIE=NOXTIE+1
      GO TO 90127
90127 NTIES=NTIES+1
90128 CONTINUE
      IF(NOXTIE)90129,90129,1
1     IND=IND+1
90129 IF(NOYTIE)2,2,3
3     IND=IND+1
2     IF(IND)90134,90134,90132
90132 TIINDE(NCELL)=TIINDE(NCELL)+NTIES
C  SUBROUTINE DO TIE REPLACES NEXT STATEMENT
      CALL DOTIES(BEGIN,END,NOXTIE,NOYTIE,NOOFTI)
90134 CONTINUE
      RETURN
      END
      SUBROUTINE EXPO(SAMSIZ)
      INTEGERSAMSIZ,BATCH
      INTEGER BASSET
      REAL MEAN
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IUNRNK(40)
     1,BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN(5)
      COMMON MEAN,VAR
      SCOMAX=1.0
      IBASE(1)=1009666569
      IBASE(2)=1291401631
      MEAN= 0.500
      RM=MEAN*2.0
      AM=MEAN
      DO 1 J=1,2
      BASSE =((J-1)*IBASE(J))+((2-J)*IBASE(J))
      CALL SETBAS(BASSE)
      DO 2 I=1,SAMSIZ
      DO 2 K=1,NOSAMS
90001 R      (K,I,J)=(-1.0)*AM*ALOG(RAND(SCOMAX))
      IF(R(K,I,J)-RM)4,4,90001
4     CONTINUE
C
C
  2   CONTINUE
  1   CONTINUE
      RETURN
      END
      SUBROUTINE RNDSCO(SAMSIZ)
      INTEGERSAMSIZ,BATCH
      INTEGERTIINDE,TIEGRU,BEGIN,END,BEGNTI,ENDTIE
      INTEGERTOTTIE
      INTEGERTTIIN
      REALIUNRNK
      INTEGER BASSET
      REAL MEAN
      REAL NORM
      DIMENSIONNOOFCL(5),TOTTIE(5)
C
      COMMON IRNSTR,POS(40)
      COMMONNOSAMS
      COMMON R(100,20,2),U(100,5),IBASE(2),JBASE(2),XUNIY(40),IU
     1NRNK(40),BEGNTI(20),ENDTIE(20),TIINDE(5),TTIIN (5),NOOFTI(5)
      COMMON MEAN,VAR
      DO 1 J=1,2
      BASSET=((J-1)*IBASE(J))+((2-J)*IBASE(J))
      CALL SETBAS(BASSET)
      DO 5 I=1,SAMSIZ
      DO 2 K=1,NOSAMS
    4 N = RAND(2.) + 1.                                                 OS/
      ETA = SQRT(ALOG(1./RAND(.5)**2))                                  OS/
      X = (ETA - (2.515517 + ETA*(.802853 + .010328 * ETA))
     1/ (1. + ETA*(1.432788 + ETA*(.189269 + .001308*ETA)))
     2)*(-1.)**N
      R(K,I,J)=MEAN+VAR*X
      IF(ABS(R(K,I,J)).GT.5)GO TO 4
    2 CONTINUE
    5 CONTINUE
      CALL SAVBAS(JBASE(J))
    1 CONTINUE
      RETURN
      END
```

Table 1

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from normal distribution

2. 5000 pairs of samples of size 5

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-5.04	.0002	.0000	.0004	.0006	.0000	.0000	.0000	.0000
.0005	-3.36	.0081	.0075	.0077	.0089	.0058	.0069	.0002	.0070
.01	-2.90	.0143	.0152	.0144	.0153	.0066	.0070	.0002	.0070
.025	-2.31	.0364	.0373	.0364	.0354	.0299	.0329	.0030	.0346
.05	-1.86	.0683	.0682	.0710	.0705	.0655	.0638	.0211	.0653
.10	-1.40	.1283	.1281	.1300	.1315	.1236	.1263	.0473	.1253
.20	- .89	.2496	.2522	.2535	.2499	.2144	.2109	.2436	.2082
.25	- .71	.3059	.3069	.3066	.3026	.3144	.3207	.2623	.3153
.75	.71	.7973	.7962	.7993	.7901	.7873	.8009	.7953	.8039
.80	.89	.8387	.8397	.8417	.8449	.8565	.8475	.9446	.8488
.90	1.40	.9200	.9223	.9219	.9230	.9248	.9276	.9711	.9258
.95	1.86	.9608	.9605	.9588	.9604	.9692	.9633	.9974	.9623
.975	2.31	.9799	.9800	.9797	.9790	.9842	.9807	.9988	.9804
.99	2.90	.9914	.9910	.9905	.9901	.9934	.9912	1.00	.9914
.995	3.36	.9955	.9955	.9952	.9949	.9942	.9914	1.00	.9914
.9995	5.04	.9994	.9997	.9994	.9989	.9992	.9994	1.00	1.00

Table 2

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from normal distribution

2. 5000 pairs of samples of size 10

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-5.04	.0012	.0010	.0010	.0008	.0007	.0006	.0000	.0007
.005	-3.36	.0090	.0086	.0094	.0074	.0069	.0058	.0006	.0054
.01	-2.90	.0156	.0158	.0159	.0170	.0180	.0181	.0030	.0190
.025	-2.31	.0345	.0354	.0345	.0373	.0362	.0380	.0131	.0386
.05	-1.86	.0678	.0652	.0657	.0639	.0660	.0732	.0337	.0768
.10	-1.40	.1315	.1312	.1324	.1296	.1222	.1194	.0956	.1134
.20	- .89	.2519	.2498	.2499	.2577	.2573	.2418	.2567	.2350
.25	- .71	.3091	.3053	.3039	.3141	.3002	.3347	.3248	.3454
.75	.71	.8040	.8056	.8050	.8061	.8045	.7948	.7649	.7868
.80	.89	.8417	.8430	.8406	.8396	.8426	.8509	.8620	.8544
.90	1.40	.9293	.9301	.9282	.9277	.9275	.9241	.9711	.9243
.95	1.86	.9639	.9633	.9651	.9670	.9636	.9634	.9921	.9648
.975	2.31	.9817	.9803	.9814	.9823	.9828	.9807	.9968	.9808
.99	2.90	.9917	.9916	.9911	.9917	.9931	.9934	.9998	.9932
.995	3.36	.9957	.9960	.9953	.9950	.9954	.9960	.9999	.9960
.9995	5.04	.9994	.9994	.9994	.9987	.9990	.9993	1.0000	.9990

Table 3

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from normal distribution

2. 5000 pairs of samples of size 20

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-3.55	.0010	.0011	.0007	.0006	.0006	.0008	.0000	.0006
.005	-2.70	.0080	.0077	.0079	.0072	.0073	.0079	.0033	.0082
.01	-2.42	.0145	.0147	.0143	.0133	.0118	.0141	.0063	.0151
.025	-2.02	.0342	.0342	.0325	.0321	.0317	.0351	.0386	.0365
.05	-1.68	.0670	.0676	.0646	.0677	.0635	.0722	.0702	.0755
.10	-1.30	.1303	.1272	.1293	.1448	.1304	.1319	.1233	.1321
.20	- .85	.2468	.2450	.2485	.2487	.2445	.2568	.2515	.2644
.25	- .68	.3049	.3047	.3040	.3132	.3109	.3028	.3480	.2958
.75	.68	.8180	.8151	.8155	.8180	.8037	.8065	.7976	.8120
.80	.85	.8578	.8590	.8542	.8559	.8458	.8572	.8624	.8631
.90	1.30	.9294	.9305	.9300	.9283	.9300	.9336	.9373	.9313
.95	1.68	.9668	.9668	.9668	.9651	.9644	.9694	.9675	.9685
.975	2.02	.9844	.9834	.9837	.9829	.9834	.9854	.9943	.9861
.99	2.42	.9942	.9938	.9944	.9926	.9932	.9936	.9980	.9936
.995	2.70	.9967	.9968	.9971	.9973	.9959	.9968	.9994	.9967
.9995	3.55	.9996	.9996	.9998	.9996	.9992	.9994	1.0000	.9994

Table 4

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 5

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-5.04	.0008	.0006	.0004	.0002	.0002	.0008	.0004	.0000
.005	-3.36	.0088	.0092	.0088	.0092	.0091	.0086	.0085	.0104
.01	-2.90	.0157	.0153	.0153	.0156	.0152	.0156	.0148	.0104
.025	-2.31	.0349	.0350	.0343	.0353	.0356	.0347	.0329	.0441
.05	-1.86	.0719	.0713	.0710	.0698	.0701	.0721	.0654	.0662
.10	-1.40	.1291	.1296	.1278	.1278	.1300	.1272	.1232	.1341
.20	- .89	.2539	.2552	.2561	.2566	.2553	.2534	.2590	.1860
.25	- .71	.3151	.3143	.3161	.3148	.3145	.3207	.2962	.3299
.75	.71	.8101	.8111	.8108	.8136	.8092	.8104	.8109	.8159
.80	.89	.8531	.8521	.8523	.8494	.8488	.8564	.8626	.8277
.90	1.40	.9266	.9269	.9267	.9283	.9248	.9238	.9310	.9326
.95	1.86	.9627	.9629	.9631	.9635	.9612	.9639	.9618	.9589
.975	2.31	.9812	.9810	.9825	.9820	.9799	.9809	.9795	.9728
.99	2.90	.9930	.9927	.9928	.9918	.9912	.9918	.9897	.9728
.995	3.36	.9957	.9958	.9954	.9953	.9961	.9951	.9956	.9728
.9995	.04	.9995	.9993	.9991	.9990	.9989	.9982	.9988	.9800

Table 5

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from rectangular distribution

2. 5000 pairs of samples of size 10

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-3.92	.0002	.0002	.0002	.0002	.0000	.0004	.0002	.0016
.005	-2.88	.0050	.0050	.0048	.0053	.0055	.0059	.0042	.0067
.01	-2.55	.0101	.0102	.0103	.0107	.0103	.0115	.0074	.0120
.025	-2.10	.0252	.0256	.0259	.0274	.0259	.0248	.0224	.0234
.05	-1.73	.0483	.0476	.0485	.0490	.0505	.0510	.0490	.0621
.10	-1.33	.0994	.0996	.0999	.1014	.0998	.0989	.0944	.1342
.20	- .86	.1998	.1996	.1995	.2003	.2017	.2066	.1912	.2191
.25	- .69	.2477	.2486	.2498	.2504	.2479	.2550	.2430	.2527
.75	.69	.7535	.7542	.7484	.7531	.7554	.7555	.7571	.7637
.80	.86	.8051	.8052	.8006	.8038	.8008	.8017	.8066	.7963
.90	1.33	.9019	.9022	.8963	.9006	.8987	.8994	.9027	.8734
.95	1.73	.9506	.9498	.9474	.9505	.9503	.9519	.9513	.9433
.975	2.10	.9758	.9759	.9716	.9756	.9754	.9747	.9741	.9760
.99	2.55	.9893	.9895	.9857	.9899	.9920	.9890	.9894	.9876
.995	2.88	.9942	.9942	.9907	.9945	.9960	.9940	.9931	.9944
.9995	3.92	.9989	.9987	.9951	.9989	.9987	.9993	.9989	.9984

Table 6

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 20

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-3.55	.0007	.0012	.0012	.0011	.0011	.0013	.0012	.0014
.005	-2.70	.0059	.0060	.0056	.0064	.0061	.0065	.0060	.0075
.01	-2.42	.0115	.0116	.0119	.0120	.0113	.0118	.0087	.0091
.025	-2.02	.0271	.0269	.0274	.0256	.0265	.0270	.0237	.0282
.05	-1.68	.0513	.0512	.0516	.0528	.0532	.0521	.0501	.0555
.10	-1.30	.0986	.0984	.0992	.1016	.0997	.1034	.1006	.0874
.20	- .85	.1981	.1982	.1981	.2001	.2014	.2013	.2015	.2213
.25	- .68	.2502	.2499	.2483	.2476	.2478	.2502	.2456	.2224
.75	.68	.7495	.7513	.7476	.7509	.7528	.7507	.7566	.7870
.80	.85	.7985	.7986	.7970	.7982	.7973	.8043	.7998	.7890
.90	1.30	.8955	.8959	.8947	.8941	.8962	.8961	.8977	.9137
.95	1.68	.9455	.9447	.9441	.9453	.9445	.9469	.9455	.9467
.975	2.02	.9731	.9724	.9736	.9743	.9744	.9764	.9732	.9741
.99	2.42	.9886	.9880	.9936	.9888	.9884	.9898	.9908	.9910
.995	2.70	.9948	.9944	.9944	.9940	.9940	.9948	.9959	.9931
.9995	3.55	.9996	.9996	.9994	.9994	.9996	.9992	.9996	.9988

Table 7

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from exponential distribution

2. 5000 pairs of samples of size 5

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-5.04	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.005	-3.36	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.01	-2.90	.0000	.0000	.0000	.0000	.0077	.0000	.0000	.0100
.025	-2.31	.0245	.0090	.0145	.0245	.0290	.0281	.0245	.0545
.05	-1.86	.0750	.0800	.0750	.0650	.0850	.0950	.0500	.0700
.10	-1.40	.1459	.1459	.1518	.1477	.1554	.1318	.1059	.1036
.20	- .89	.2254	.2345	.2263	.2554	.2472	.2545	.2063	.2100
.25	- .71	.2882	.2982	.3100	.3036	.3154	.3109	.2536	.2827
.75	.71	.7913	.7968	.7813	.7813	.7659	.7990	.7627	.7427
.80	.89	.8313	.8218	.8222	.8227	.8227	.8318	.8409	.8204
.90	1.40	.8972	.8972	.8972	.8972	.9109	.9018	.9009	.8809
.95	1.86	.9490	.9436	.9345	.9236	.9390	.9545	.9290	.9700
.975	2.31	.9700	.9700	.9750	.9650	.9750	.9750	.9650	.9850
.99	2.90	.9800	.9800	.9800	.9800	.9822	.9900	.9822	1.0000
.995	3.36	.9800	.9800	.9800	.9800	.9900	.9900	1.0000	1.0000
.9995	5.04	1.0000	1.0000	1.0000	1.0000	1.0000	.9900	1.0000	1.0000

Table 8

Empirical Relative Frequency Distribution of the Student t Statistic

Conditions: 1. Sampling from exponential distribution

2. 5000 pairs of samples of size 20

Critical Probability	Point with Probability Equal to or less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.0005	-5.04	.0005	.0004	.0004	.0003	.0001	.0001	.0000	.0000
.005	-3.36	.0076	.0073	.0077	.0070	.0069	.0082	.0060	.0014
.01	-2.90	.0162	.0152	.0151	.0154	.0143	.0145	.0098	.0020
.025	-2.31	.0368	.0372	.0365	.0373	.0371	.0365	.0382	.0256
.05	-1.86	.0734	.0718	.0741	.0748	.0733	.0701	.0738	.0568
.10	-1.40	.1414	.1399	.1404	.1419	.1419	.1327	.1439	.1180
.20	- .89	.2669	.2661	.2675	.2670	.2632	.2657	.2712	.2677
.25	- .71	.3210	.3221	.3247	.3205	.3201	.3214	.3274	.3441
.75	.71	.8050	.8045	.8073	.8069	.8104	.8077	.8066	.7660
.80	.89	.8461	.8464	.8478	.8503	.8496	.8448	.8475	.8719
.90	1.40	.9276	.9264	.9278	.9277	.9265	.9285	.9287	.9594
.95	1.86	.9651	.9574	.9645	.9641	.9654	.9656	.9627	.9836
.975	2.31	.9859	.9858	.9854	.9857	.9846	.9845	.9882	.9990
.99	2.90	.9944	.9944	.9942	.9946	.9960	.9956	.9958	1.0000
.995	3.36	.9978	.9981	.9982	.9982	.9980	.9979	.9984	1.0000
.9995	5.04	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	1.0000	1.0000

Table 9

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 5
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0136	.0144	.0158	.0178	.0216	.0274	.0378	.0920
.050	2	.0290	.0286	.0300	.0344	.0500	.0564	.0824	.1910
.100	4	.0934	.0948	.0972	.1040	.1340	.1612	.2218	.4384
.200	5	.1540	.1546	.1570	.1616	.1998	.2284	.2898	.4550
.500		.4999	.5009	.5031	.5216	.5647	.5978	.6650	.8064
.800		.7999	.8051	.8062	.8113	.8332	.8434	.8655	.8990
.900		.8999	.9035	.9020	.9039	.9161	.9211	.9289	.9361
.950		.9499	.9517	.9510	.9519	.9580	.9605	.9644	.9680

Table 10

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 5
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0136	.0146	.0156	.0156	.0138	.0162	.0138	.0172
.050	2	.0290	.0284	.0278	.0290	.0284	.0266	.0278	.0360
.100	4	.0934	.0938	.0940	.0914	.0932	.0854	.0846	.0858
.200	5	.1540	.1548	.1516	.1492	.1492	.1410	.1402	.1452
.500		.4999	.4973	.4968	.4936	.4925	.4918	.4734	.4672
.800		.7999	.8013	.7979	.7986	.7946	.8026	.7903	.8002
.900		.8999	.9012	.8987	.8988	.8960	.9022	.8939	.9025
.950		.9499	.9506	.9493	.9494	.9478	.9511	.9469	.9512

Table 11

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 5
3. Ties handled by assigning members of a tie group the mean of the ranks for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0136	.0144	.0158	.0178	.0216	.0274	.0358	.0844
.050	2	.0290	.0286	.0300	.0344	.0500	.0584	.0852	.1880
.100	4	.0934	.0948	.0972	.1040	.1340	.1261	.1727	.3491
.200	5	.1540	.1546	.1570	.1616	.1998	.1838	.2425	.4429
.500		.4999	.5009	.5031	.5216	.5647	.5330	.6110	.7410
.800		.7999	.8051	.8062	.8113	.8332	.7606	.8066	.9002
.900		.8999	.9035	.9020	.9039	.9161	.8752	.8862	.9002
.950		.9499	.9517	.9510	.9519	.9580	.8752	.8862	.9002

Table 12

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution.
2. 5000 pairs of samples of size 10.
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0006	.0006	.0006	.0010	.0016	.0036	.0082	.0544
.002	10	.0010	.0006	.0006	.0016	.0048	.0092	.0156	.0906
.005	13	.0032	.0034	.0040	.0048	.0114	.0190	.0354	.1622
.010	16	.0082	.0086	.0082	.0124	.0282	.0426	.0802	.2760
.020	19	.0180	.0182	.0192	.0256	.0510	.0744	.1330	.3244
.050	23	.0386	.0396	.0440	.0564	.1000	.1396	.2220	.4498
.100	27	.0876	.0882	.0908	.1110	.1862	.2428	.3496	.6086
.200	32	.1874	.1908	.1954	.2200	.3198	.3682	.5074	.7782
.500	41	.5124	.5153	.5228	.5473	.6335	.6858	.7712	.9028
.800	47	.8153	.8163	.8144	.8279	.8618	.8843	.9179	.9698
.900	48	.8578	.8602	.8575	.8722	.8961	.9071	.9338	.9709
.950	49	.9541	.9557	.9536	.9572	.9618	.9679	.9855	.9959

Table 13

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 10
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0008	.0008	.0006	.0006	.0006	.0008	.0006	.0006
.002	10	.0018	.0018	.0014	.0012	.0012	.0018	.0014	.0014
.005	13	.0042	.0040	.0038	.0038	.0036	.0034	.0040	.0030
.010	16	.0082	.0088	.0082	.0084	.0080	.0090	.0086	.0070
.020	19	.0186	.0190	.0184	.0178	.0172	.0178	.0164	.0176
.050	23	.0436	.0444	.0436	.0440	.0442	.0388	.0424	.0392
.100	27	.0846	.0822	.0836	.0840	.0846	.0878	.0856	.0800
.200	32	.1886	.1892	.1880	.1796	.1862	.1880	.1838	.1858
.500	41	.4999	.4972	.4965	.4992	.4947	.4960	.5036	.4955
.800	47	.7999	.7979	.8032	.8064	.8066	.8050	.8053	.8046
.900	48	.8437	.8442	.8539	.8556	.8536	.8484	.8498	.8504
.950	49	.9499	.9515	.9519	.9560	.9514	.9496	.9486	.9514

Table 14

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution.
2. 5000 pairs of samples of size 10.
3. Ties handled by assigning members of a tie group the mean of the ranks for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0008	.0008	.0006	.0010	.0002	.0006	.0002	.0000
.002	10	.0018	.0020	.0016	.0010	.0010	.0008	.0006	.0004
.005	13	.0042	.0042	.0038	.0032	.0028	.0020	.0024	.0004
.010	16	.0082	.0092	.0080	.0076	.0064	.0060	.0054	.0026
.020	19	.0186	.0190	.0190	.0188	.0150	.0142	.0134	.0026
.050	23	.0436	.0454	.0434	.0416	.0400	.0322	.0290	.0126
.100	27	.0846	.0858	.0870	.0870	.0842	.0758	.0654	.0426
.200	32	.1886	.1942	.1924	.1852	.1746	.1764	.1578	.1208
.500	41	.5186	.5282	.5256	.5184	.5168	.5086	.4952	.4936
.800	47	.8396	.8580	.8580	.8614	.8492	.8632	.8720	.8200
.900	48	.9042	.9228	.9228	.9182	.9176	.9160	.8986	.8200
.950	49	.9700	.9862	.9860	.9828	.9792	.9726	.9430	.8200

Table 15

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 20
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with P Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0015	.0015	.0017	.0030	.0160	.0255	.0712	.3134
.002	88	.0034	.0030	.0028	.0068	.0254	.0418	.1088	.4364
.005	97	.0073	.0065	.0093	.0147	.0441	.0734	.1692	.5349
.010	105	.0140	.0147	.0167	.0250	.0729	.1165	.2339	.6071
.020	114	.0278	.0282	.0306	.0440	.1164	.1784	.3223	.6974
.050	127	.0620	.0619	.0690	.0945	.2050	.2925	.4708	.8156
.100	136	.1016	.1042	.1136	.1512	.2728	.3854	.5690	.8636
.200	151	.2182	.2189	.2330	.2831	.4313	.5628	.7272	.9355
.500	172	.4999	.5018	.5137	.5560	.6818	.7783	.8857	.9765
.800	187	.7945	.7910	.7970	.8177	.8649	.8995	.9616	.9900
.900	192	.8945	.8939	.9014	.9125	.9217	.9607	.9828	.9955
.950	195	.9499	.9495	.9543	.9592	.9516	.9811	.9907	.9970

Table 16

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution.
2. 5000 pairs of samples of size 20.
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0015	.0014	.0014	.0015	.0016	.0015	.0018	.0009
.002	88	.0034	.0034	.0034	.0030	.0032	.0032	.0032	.0028
.005	97	.0073	.0069	.0070	.0080	.0078	.0066	.0075	.0066
.010	105	.0140	.0145	.0154	.0142	.0158	.0141	.0140	.0122
.020	114	.0278	.0276	.0276	.0275	.0280	.0263	.0262	.0235
.050	127	.0620	.0627	.0618	.0635	.0637	.0627	.0579	.0574
.100	136	.1016	.1034	.1022	.1010	.1084	.0996	.1006	.0992
.200	151	.2182	.2178	.2173	.2152	.2263	.2171	.2173	.2231
.500	172	.4999	.4965	.4995	.5010	.4993	.5002	.4955	.5037
.800	187	.7945	.7918	.7935	.7940	.8010	.7881	.7882	.7929
.900	192	.8945	.8944	.8987	.8993	.8994	.8950	.8970	.9013
.950	195	.9499	.9509	.9526	.9529	.9520	.9505	.9529	.9526

Table 17

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from rectangular distribution
2. 5000 pairs of samples of size 20
3. Ties handled by assigning members of a tie group the mean of the ranks for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0015	.0014	.0019	.0017	.0020	.0012	.0019	.0000
.002	88	.0034	.0032	.0030	.0032	.0030	.0016	.0024	.0008
.005	97	.0073	.0076	.0068	.0062	.0058	.0053	.0057	.0014
.010	105	.0140	.0150	.0145	.0142	.0128	.0102	.0093	.0045
.020	114	.0278	.0276	.0278	.0271	.0254	.0207	.0411	.0078
.050	127	.0620	.0636	.0625	.0631	.0600	.0549	.0716	.0348
.100	136	.1016	.1032	.1034	.0996	.1000	.0902	.1040	.0404
.200	151	.2182	.2204	.2192	.2160	.2141	.2049	.2172	.1588
.500	172	.4999	.5026	.4998	.5012	.4977	.4960	.4991	.4308
.800	187	.7945	.7956	.7972	.8004	.7932	.7868	.7942	.7963
.900	192	.8945	.8988	.9021	.9052	.9024	.9048	.9309	.8678
.950	195	.9499	.9555	.9562	.9580	.9555	.9530	.9689	.8678

Table 18

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from normal distribution.
2. 5000 pairs of samples of size 5
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0200	.0200	.0200	.0300	.0400	.0300	.0600	.2400
.050	2	.0300	.0300	.0200	.0300	.0500	.1000	.1200	.2800
.100	4	.0800	.1000	.1200	.1200	.2100	.2600	.3300	.5400
.200	5	.1800	.1900	.1800	.2200	.2900	.3500	.4500	.6500
.500	9	.5000	.5200	.5700	.5900	.6300	.6600	.7300	.8200
.800	11	.7999	.7964	.7799	.7505	.8241	.8788	.9123	.9900
.900	12	.8999	.8999	.8857	.8571	.9071	.9285	.9499	.9928
.950	12	.9499	.9499	.9428	.9285	.9535	.9642	.9749	.9964

Table 19

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions: 1. Sampling from exponential distribution.

2. 5000 pairs of samples of size 5.

3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution						
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0200	.0200	.0200	.0254	.0224	.0182	.0242	.0236
.050	2	.0300	.0300	.0246	.0322	.0344	.0296	.0474	.0420
.100	4	.0800	.0908	.0920	.1020	.0906	.0926	.0890	.1144
.200	5	.1800	.1936	.1682	.1754	.1696	.1590	.1440	.1804
.500		.5000	.5196	.5354	.5212	.5598	.5358	.5542	.5788
.800		.7899	.8038	.8031	.7607	.7855	.7796	.7790	.8095
.900		.8899	.9052	.9067	.8737	.8824	.8839	.8721	.8994
.950		.9399	.9536	.9533	.9368	.9412	.9419	.9260	.9497

Table 20

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution.
2. 5000 pairs of samples of size 5.
3. Ties handled by assigning members of a tie group the mean of the ranks for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.020	.0200	.0200	.0200	.0100	.0100	.0000	.0000
.050	2	.0300	.0300	.0200	.0200	.0300	.0100	.0000	.0000
.100	4	.0800	.0800	.0700	.0900	.0400	.0300	.0400	.0100
.200	5	.1800	.1800	.1300	.1300	.1200	.1100	.1100	.0900
.500		.5000	.4900	.4900	.4800	.4800	.4800	.4100	.3000
.800		.7999	.7882	.7982	.7523	.7852	.7688	.6894	.7100
.900		.8999	.9185	.9099	.8957	.9057	.8985	.7742	.7100
.950		.9499	.9292	.9349	.9278	.9378	.9342	.7921	.7100

Table 21

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution.
2. 5000 pairs of samples of size 10.
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0000	.0000	.0000	.0000	.0000	.0100	.0300	.1300
.002	10	.0000	.0000	.0000	.0000	.0000	.0200	.0600	.1900
.005	13	.0000	.0000	.0000	.0000	.0000	.0400	.0900	.2800
.010	16	.0100	.0100	.0100	.0200	.0600	.0600	.1800	.4500
.020	19	.0200	.0200	.0300	.0300	.0900	.0800	.2600	.5400
.050	23	.0600	.0500	.0400	.0500	.1600	.2000	.3600	.6200
.100	27	.1200	.1000	.1000	.1100	.2600	.3200	.5300	.8000
.200	32	.2100	.2400	.2700	.2800	.4200	.5300	.6300	.9200
.500	40	.4999	.4957	.5014	.6042	.7000	.7014	.8314	.9714
.800	47	.7999	.8185	.8557	.8514	.8828	.9228	.9514	.9900
.900	48	.9000	.9200	.9400	.9100	.9600	.9700	.9700	1.0000
.950	49	.9199	.9342	.9542	.9185	.9628	.9728	.9728	1.0000

Table 22

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution.
2. 5000 pairs of samples of size 10.
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0000	.0000	.0000	.0000	.0000	.0006	.0000	.0006
.002	10	.0000	.0000	.0000	.0000	.0000	.0018	.0004	.0022
.005	13	.0000	.0034	.0076	.0000	.0000	.0024	.0022	.0054
.010	16	.0100	.0100	.0100	.0020	.0006	.0068	.0076	.0140
.020	19	.0200	.0200	.0156	.0154	.0104	.0154	.0136	.0292
.050	23	.0600	.0668	.0628	.0648	.0396	.0446	.0398	.0580
.100	27	.1200	.1100	.1224	.1248	.0938	.0928	.0868	.1076
.200	32	.2100	.2114	.2136	.2256	.2110	.1838	.1888	.2046
.500	40	.4999	.4778	.4989	.4753	.4809	.4492	.4473	.4551
.800	47	.7999	.8209	.8366	.8337	.8272	.8158	.8171	.8055
.900	48	.9000	.9100	.9212	.9212	.9202	.9190	.9176	.9096
.950	49	.9199	.9269	.9361	.9358	.9358	.9340	.9331	.9272

Table 23

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution
2. 5000 pairs of samples of size 10
3. Ties handled by assigning members of a tie group the mean of the ranks for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution						
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.002	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.005	13	.0000	.0000	.0100	.0000	.0000	.0000	.0000	.0000
.010	16	.0100	.0100	.0100	.0000	.0000	.0000	.0000	.0000
.020	19	.0200	.0200	.0100	.0100	.0000	.0100	.0000	.0000
.050	23	.0600	.0700	.0600	.0600	.0200	.0400	.0100	.0100
.100	27	.1200	.1300	.1300	.1200	.0700	.0900	.0600	.0400
.200	32	.2100	.2200	.2300	.2400	.1900	.1400	.1300	.1000
.500	40	.5600	.5400	.5700	.5400	.5100	.4600	.4000	.4600
.800	47	.8600	.8700	.8700	.8500	.8500	.8500	.8400	.5600
.900	48	.9000	.9300	.9300	.8596	.9000	.9500	.9300	.5600
.950	49	.9700	.9900	.9900	1.00	.9900	1.00	.9700	.5600

Table 24

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution.
2. 5000 pairs of samples of size 20.
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0100	.0025	.0000	.0000	.0300	.0600	.1800	.5900
.002	88	.0100	.0100	.0100	.0100	.0400	.0700	.2400	.7000
.005	97	.0100	.0100	.0100	.0400	.0675	.1300	.3100	.8025
.010	105	.0100	.0100	.0175	.0500	.1050	.2550	.3725	.8525
.020	114	.0350	.0350	.0500	.0700	.1600	.3150	.4700	.8700
.050	127	.0900	.1050	.1000	.1050	.3200	.4725	.6700	.9350
.100	136	.1200	.1300	.1400	.1400	.4100	.5600	.7400	.9600
.200	151	.2300	.2175	.2400	.3550	.5875	.7150	.8850	.9800
.500	173	.4999	.5228	.5742	.6299	.8128	.8971	.9514	1.0000
.800	187	.8000	.8025	.8250	.8400	.9325	.9800	.9900	1.0000
.900	193	.8999	.9099	.9011	.9366	.9822	1.0000	.9911	1.0000
.950	194	.9199	.9299	.9233	.9499	.9866	1.0000	.9933	1.0000

Table 25

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution
2. 5000 pairs of samples of size 20
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0100	.0100	.0088	.0095	.0031	.0006	.0025	.0006
.002	88	.0100	.0100	.0100	.0100	.0062	.0030	.0052	.0016
.005	97	.0100	.0100	.0102	.0103	.0131	.0069	.0101	.0071
.010	105	.0100	.0105	.0183	.0174	.0202	.0154	.0161	.0149
.020	114	.0350	.0429	.0442	.0358	.0337	.0303	.0283	.0269
.050	127	.0900	.0939	.0998	.0861	.0721	.0623	.0645	.0654
.100	136	.1200	.1276	.1240	.1252	.1114	.0964	.1064	.1126
.200	151	.2300	.2230	.2226	.2045	.2088	.2145	.2247	.2424
.500	173	.4999	.4951	.5069	.5066	.5063	.5401	.5222	.5459
.800	187	.8000	.8030	.7906	.8060	.8116	.8250	.7964	.8088
.900	193	.8999	.9068	.9011	.9129	.9208	.9221	.9121	.9180
.950	194	.9199	.9269	.9221	.9320	.9381	.9395	.9313	.9364

Table 26

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from exponential distribution.
2. 5000 pairs of samples of size 20.
3. Ties handled by assigning members of a tie group the mean of the ranks for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0100	.0100	.0100	.0100	.0025	.0000	.0000	.0000
.002	88	.0100	.0100	.0100	.0100	.0100	.0000	.0000	.0000
.005	97	.0100	.0100	.0100	.0100	.0100	.0025	.0100	.0000
.010	105	.0100	.0100	.0100	.0200	.0100	.0025	.0100	.0000
.020	114	.0350	.0400	.0400	.0450	.0200	.0100	.0100	.0000
.050	127	.0900	.0900	.0900	.1000	.0600	.0425	.0400	.0175
.100	136	.1200	.1300	.1300	.1200	.1000	.0700	.0700	.0200
.200	151	.2300	.2300	.2300	.2348	.1850	.1600	.1975	.1400
.500	173	.4999	.5071	.5071	.5212	.4928	.4871	.4642	.4100
.800	187	.8000	.8025	.8025	.8098	.8450	.8500	.7525	.7750
.900	193	.8999	.9199	.9199	.8842	.9366	.9255	.8733	.8400
.950	194	.9199	.9399	.9399	.9131	.9499	.9366	.8999	.8400

Table 27

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from normal distribution.
2. 5000 pairs of samples of size 5.
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0186	.0196	.0222	.0298	.1068	.1666	.5912	.1774
.050	2	.0334	.0352	.0432	.0648	.1322	.2242	.5934	.2386
.100	4	.1044	.1040	.1112	.1560	.3246	.4842	.7022	.5154
.200	5	.1586	.1572	.1672	.2170	.4034	.5524	.9108	.5784
.500		.4999	.5044	.5116	.5667	.7217	.8354	.9498	.8447
.800		.7999	.7981	.8068	.8315	.8851	.9342	.9958	.9376
.900		.8999	.8996	.9053	.9171	.9409	.9600	.9973	.9603
.950		.9499	.9498	.9546	.9585	.9704	.9800	.9986	.9801

Table 28

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from Normal Distribution.
2. 5000 pairs of samples of size 5.
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001									
.002									
.005									
.010	0								
.020	1	.0186	.0186	.0182	.0202	.0140	.0182	.0188	.0166
.050	2	.0326	.0332	.0330	.0352	.0294	.0366	.0356	.0368
.100	4	.1012	.0984	.1010	.1010	.0826	.0932	.0970	.0952
.200	5	.1518	.1522	.1566	.1518	.1378	.1520	.1516	.1526
.500		.5084	.5083	.4999	.5022	.4800	.4959	.5065	.4912
.800		.7991	.8017	.7992	.7996	.7920	.8029	.8068	.7984
.900		.8988	.9001	.9019	.8988	.8959	.9024	.9050	.9003
.950		.9494	.9500	.9519	.9494	.9479	.9512	.9525	.9501

Table 29

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from normal distribution.
2. 5000 pairs of samples of size 10.
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0014	.0014	.0030	.0108	.0980	.1174	.4266	.1350
.002	10	.0018	.0026	.0046	.0186	.1348	.1560	.7246	.1760
.005	13	.0044	.0052	.0106	.0358	.1688	.2076	.7252	.2328
.010	16	.0114	.0126	.0196	.0596	.2988	.3480	.7676	.3822
.020	19	.0214	.0218	.0344	.0952	.3748	.4310	.8610	.4654
.050	23	.0436	.0482	.0704	.1716	.4840	.5408	.9284	.5706
.100	27	.0864	.0916	.1378	.2804	.6348	.6892	.9636	.7164
.200	32	.1866	.1952	.2560	.4286	.7776	.8276	.9892	.8486
.500	41	.5286	.5346	.5874	.7224	.9260	.9400	.9992	.9468
.800	47	.8568	.8526	.8752	.9166	.9794	.9810	1.0000	.9832
.900	48	.9114	.9088	.9260	.9512	.9906	.9912	1.00	.9936
.950	49	.9712	.9666	.9778	.9802	.9938	.9952	1.00	.9974

Table 30

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from normal distribution.
2. 5000 pairs of samples of size 10.
3. Ties handled by assigning each member of a tied group to a random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	8	.0014	.0012	.0012	.0016	.0008	.0006	.0008	.0004
.002	10	.0018	.0018	.0020	.0022	.0024	.0014	.0018	.0014
.005	13	.0044	.0048	.0046	.0038	.0046	.0046	.0032	.0030
.010	16	.0114	.0090	.0114	.0102	.0084	.0090	.0080	.0094
.020	19	.0214	.0220	.0194	.0178	.0192	.0186	.0158	.0176
.050	23	.0436	.0440	.0410	.0392	.0442	.0422	.0388	.0424
.100	27	.0864	.0846	.0854	.0778	.0908	.0886	.0844	.0900
.200	32	.1866	.1830	.1816	.1810	.1862	.1868	.1904	.1912
.500	41	.4984	.4939	.4896	.4912	.4885	.5010	.5095	.4979
.800	47	.7980	.7943	.7918	.7949	.7870	.7965	.7928	.7967
.900	48	.9004	.9024	.8982	.9021	.8938	.9012	.9029	.9002
.950	49	.9472	.9517	.9446	.9483	.9424	.9471	.9468	.9464

Table 31

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions:
1. Sampling from normal distribution.
2. 5000 pairs of samples of size 20.
3. Ties handled by putting all tied members of sample number one before those in sample number two tied for the same position.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0015	.0022	.0063	.0433	.3862	.4529	.9596	.4930
.002	88	.0030	.0042	.0112	.0704	.4810	.5742	.9762	.6240
.005	97	.0061	.0083	.0218	.1200	.5867	.6618	.9882	.7071
.010	105	.0125	.0163	.0374	.1698	.6722	.7426	.9919	.7796
.020	114	.0249	.0305	.0695	.2518	.7555	.8178	.9970	.8472
.050	127	.0563	.0681	.1394	.3830	.8510	.8991	.9983	.9177
.100	136	.0760	.1138	.2120	.4826	.8986	.9341	.9994	.9458
.200	153	.2067	.2336	.3588	.6500	.9545	.9731	.9999	.9795
.500	172	.4999	.5145	.6345	.8437	.9871	.9915	1.0000	.9924
.800	188	.7999	.8085	.8595	.9466	.9967	.9980	1.0000	.9984
.900	193	.8999	.9078	.9344	.9723	.9994	.9992	1.0000	.9998
.950	195	.9499	.9538	.9670	.9837	.9996	.9996	1.0000	.9999

Table 32

Empirical Relative Frequency Distribution of the Mann-Whitney U Statistic

Conditions: 1. Sampling from normal distribution

2. 5000 pairs of samples of size 20

3. Ties handled by assigning each member of a tied group to random number of the positions for which they are tied.

Critical Probability	Largest Value with Probability less than Critical Probability	Relative Frequencies by Number of Categories on Scale of Values of Sampled Distribution							
		Continuous	50 cells	20 cells	10 cells	5 cells	4 cells	3 cells	2 cells
.001	81	.0015	.0017	.0012	.0013	.0016	.0007	.0004	.0004
.002	88	.0030	.0036	.0026	.0024	.0032	.0016	.0010	.0008
.005	97	.0061	.0062	.0065	.0068	.0062	.0040	.0052	.0044
.010	105	.0125	.0123	.0130	.0109	.0120	.0095	.0106	.0095
.020	114	.0249	.0247	.0243	.0221	.0265	.0225	.0233	.0212
.050	127	.0563	.0564	.0546	.0585	.0593	.0574	.0565	.0529
.100	136	.0930	.0956	.0904	.0964	.1008	.1004	.0994	.0998
.200	153	.2067	.2063	.2065	.1866	.2238	.2105	.2176	.2183
.500	172	.4999	.4922	.4981	.4761	.5128	.4958	.5103	.5058
.800	188	.7999	.8012	.7940	.7802	.8042	.7996	.8131	.8097
.900	193	.8999	.8985	.8985	.8843	.9117	.8993	.9122	.9072
.950	195	.9499	.9482	.9480	.9326	.9551	.9483	.9574	.9536

DESIGN PARADIGMS

Jack N. Sparks